Mechanical Resonator I

Physics Laboratory 1

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Abstract The dynamic response of a mechanical resonator was investigated using two complementary measurement techniques: frequency-domain analysis with a lock-in amplifier and time-domain observation through a ringdown experiment. The lock-in measurement provided amplitude and phase as functions of driving frequency, revealing a broad resonance near 963.9 ± 23.7 Hz. The ringdown method, based on the exponential decay of free oscillations, yielded a resonance frequency of $f_0 = 50.04 \pm 0.03$ Hz and a very high quality factor of $Q \approx 4 \times 10^5$, indicating minimal damping and excellent energy retention. The comparison highlights the influence of excitation method and frequency range on the observed dynamics, demonstrating how steady-state and transient analyses together provide a complete characterization of mechanical resonance phenomena. These findings underline the importance of resonator quality in precision sensing and vibration control applications.

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Introduction

Mechanical resonance. A mechanical resonator is well described, to first order, by the damped harmonic oscillator model. For an effective mass m, spring constant k, and (linear) damping rate Γ, the driven motion $x(t)$ obeys

$$\ddot{x} + \Gamma\dot{x} + \omega_0^2 x = \frac{F_0}{m}\cos(\omega t), \qquad \omega_0 = \sqrt{k/m}.$$

In steady state, the displacement shows a Lorentzian-shaped amplitude response with a characteristic full width $\Delta\omega = \Gamma$ and a phase shift that evolves from 0 to π across resonance. The quality factor is $Q = \omega_0/\Gamma$. In the time domain, after switching off the drive near resonance, the oscillation amplitude decays exponentially as $x(t) \propto e^{-(\Gamma/2)t}$, which provides an independent estimate of Γ.

Scope of this experiment. In this experiment we characterize a cantilever resonator that is magnetically driven and read out with a piezoelectric sensor. From a frequency sweep around the resonance we will determine the resonance frequency $f_0 = \omega_0/2\pi$ and the quality factor Q by fitting the amplitude and phase response. A complementary ringdown measurement will be used to extract the damping rate Γ and to cross-check the value obtained from the sweep. [1]

Experiment

In this experiment a mechanical cantilever with a piezo element was used, as well as a Moku:Go and MokuOS 4.0.3. A overview of the experimental setup can be seen in Fig.1.

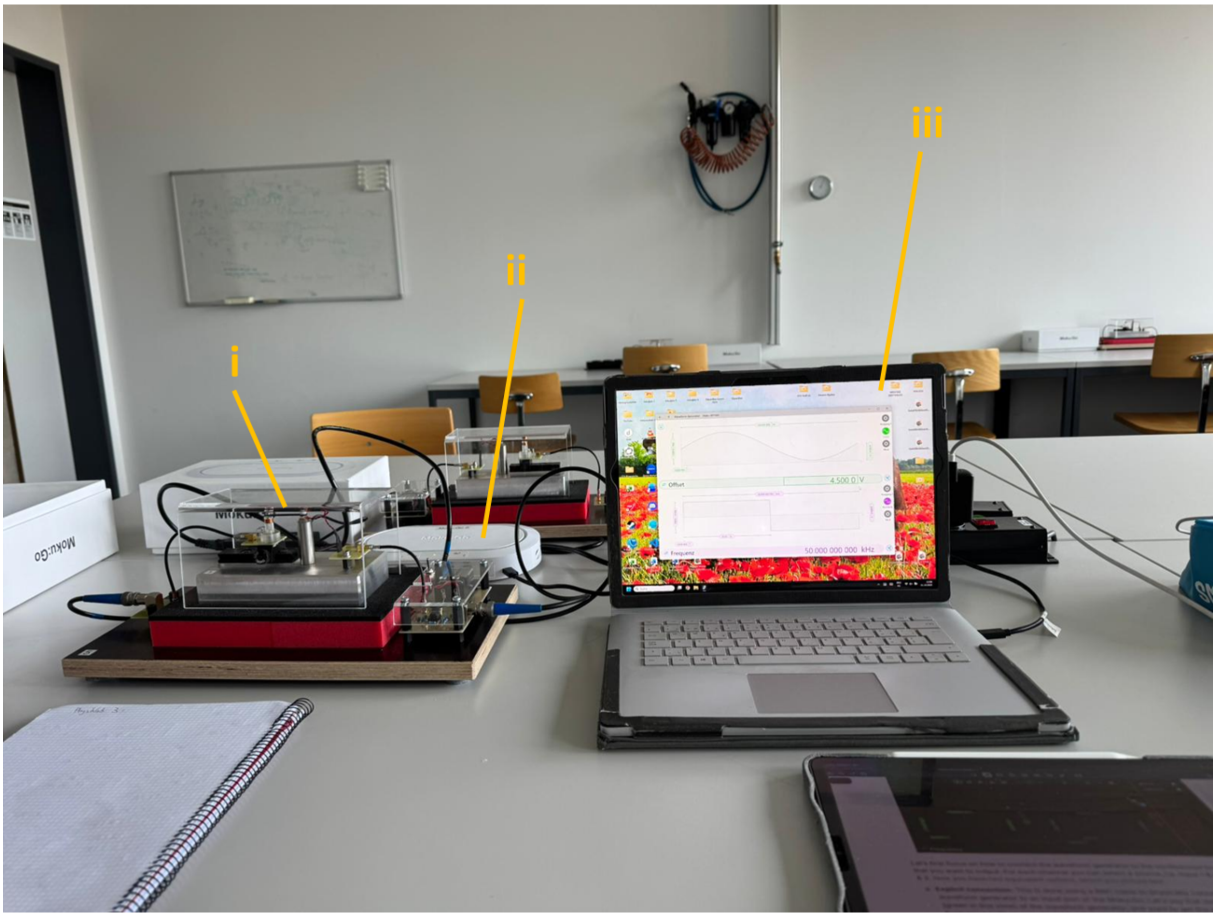

Figure 1: General set up for the experiment. On the left the cantilever and the piezo element are visible (i). In the background is the Moku:Go (ii) and on the right, a laptop with the Moku application (iii).

For the first two measurements, the output 1 and input 1 of the Moku:Go were connected, using a cable. The following measurement was conducted first with the oscilloscope and then with the lock-in amplifier (LIA). A sinus wave was generated with a frequency of 500 Hz and a peak to peak voltage of 10 V, and the output frequency, as well as the output voltage, were determined.
Then, the Moku:Go was connected to the piezo element, by connecting output 1 of the Moku:Go with the input of the cantilever, as well as connecting the output of the piezo element with input 2 of the Moku:Go, which can be seen in Fig.2, and the amplifier was turned on. A sinus wave with a peak to peak voltage of 10 V and 700 Hz was generated with the LIA and the amplitude of the piezo element was measured. The frequency was increased in steps of 50 Hz and the measurement was repeated with each increase until the frequency was 1200 Hz. Additional measurements of the amplitude were made with frequencies of 925 Hz and 975 Hz.

Figure 2: Set up for the third, forth and fifth experiment. Moku:Go output 1 (i) is connected to the input of the cantilever(ii) and the output of the piezo element (iii) is connected witch the input 2 of the Moku:Go (iv).

An automatic frequency sweep was performed with the frequency analyzer (FA), with a settling time of 100.00 ms at 2.00 ms.

Finally, a ringdown experiment was performed in the Datenlogger, seen in Fig.3, by driving the resonator at $\omega = 820$ Hz, starting the recording and muting the output of the Moku:Go. The data-points were collected in a csv datasheet.

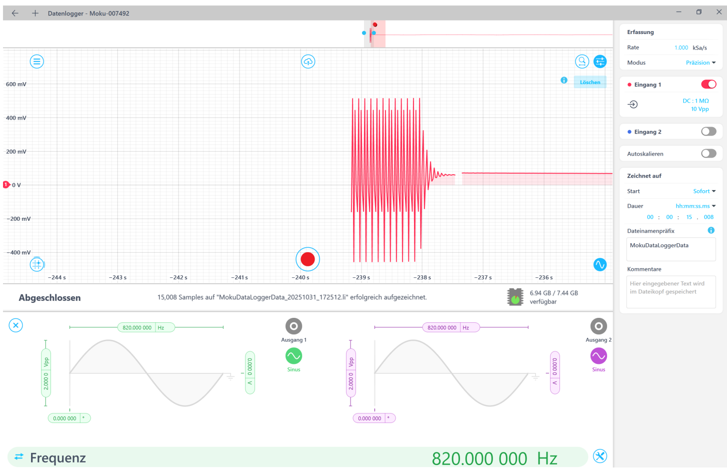

Figure 3: A screenshot of the ringdown experiment.

Results

The output frequency f_{measOS}, measured with the oscilloscope was found to be 498.7 ± 0.2 Hz, whereas the output frequency $f_{measLIA}$, measured with the lock-in amplifier was found to be 500.7 ± 0.0001 Hz. The output voltage V_{measOS}, measured with the oscilloscope was found to be 9.975 ± 0.700 V, whereas the output voltage $V_{measLIA}$, measured with the lock-in amplifier was found to be 9.972 ± 0.003 V.

The measurement of the amplitude of the piezo element can be found in the table 1 below.

Table 1: Overview of the measured results of the amplitude of the piezo element.

f / **Hz**	**X** ± δ**X** / **mV**	**Y** ± δ**Y** / **mV**
700	124.9 ± 1.8	124.8 ± 2.9
750	130.5 ± 3.4	129.6 ± 2.5
800	130.5 ± 2.7	130.5 ± 1.7
850	132.4 ± 2.5	132.4 ± 1.9
900	132.7 ± 3.2	132.5 ± 2.3
925	132.5 ± 2.9	132.9 ± 2.5
950	133.9 ± 2.8	131.9 ± 2.7
975	132.8 ± 3.3	131.4 ± 2.4
1000	132.3 ± 3.6	131.3 ± 3.0
1050	131.0 ± 3.4	130.4 ± 2.9
1100	130.7 ± 3.8	129.8 ± 3.5
1150	130.4 ± 4.1	129.8 ± 3.7
1200	128.2 ± 3.9	129.4 ± 3.4

The correct filter settings for the frequency sweep were found to be 2.00 ms with a settling time of 100.00 ms.

The data collected from the ring down experiment can be found in the data analysis Fig. 8. It has to be noted that difficulties occurred while recording the data, as the recorded data always showed much noise.

Data Analysis

The measurement with the OS was found to be faster and more easy as the measurement with the LIA. However, the signal obtained with the LIA was more stable and showed a smaller error, than the signal obtained with the OS. The relation of V_{ppOS} / V_{ppLIA} was calculated to be R = 1.0003 ± 0.0702. The following formula was used to calculate the error of the diviation R = V_{ppOS} / V_{ppLIA}:

$$\left(\frac{\Delta R}{R}\right)^2 = \left(\frac{\Delta V_{ppOS}}{V_{ppOS}}\right)^2 + \left(\frac{\Delta V_{ppLIA}}{V_{ppLIA}}\right)^2.$$

Lock-In Amplifier Measurement

The measured X and Y components of the lock-in amplifier signal were used to calculate the total magnitude A:

$$A = \sqrt{X^2 + Y^2}$$

and the phase φ:

$$\varphi = \arctan\left(\frac{Y}{X}\right).$$

The associated uncertainties were propagated from the reported instrumental uncertainties of X and Y according to standard Gaussian error propagation.
Figure 4 shows the resulting magnitude $A(f)$ as a function of the excitation frequency. The data exhibit a smooth maximum around the resonance region near $f_0 \approx 950$–$1000\,\text{Hz}$. A Lorentzian fit (Figure 6) was applied to estimate the resonance frequency and linewidth. Although the data are relatively flat, the fit gives a qualitative indication of the resonance behavior. The corresponding phase response, shown in Figure 5, remains nearly constant around 44.9 ± 0.2 °, as expected for a symmetric in-phase signal.

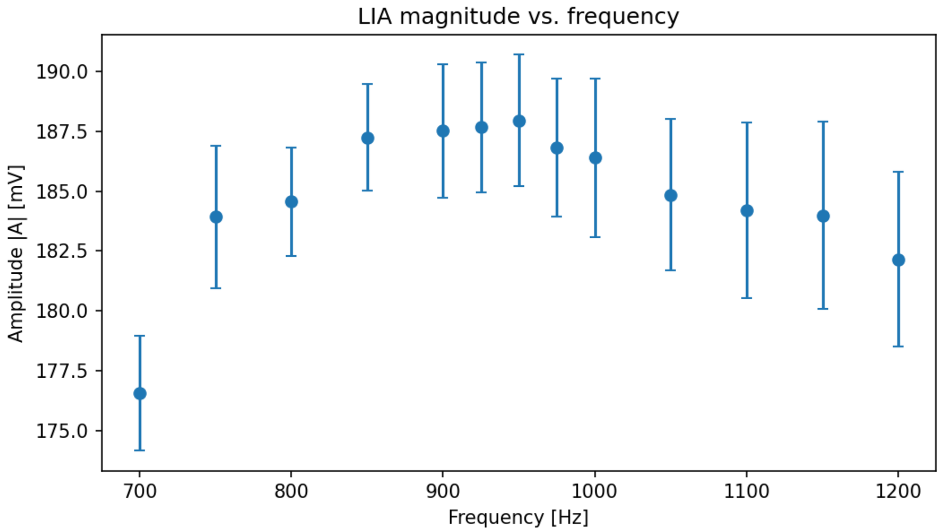

Figure 4: Measured magnitude $A(f)$ of the lock-in amplifier output with propagated uncertainties.

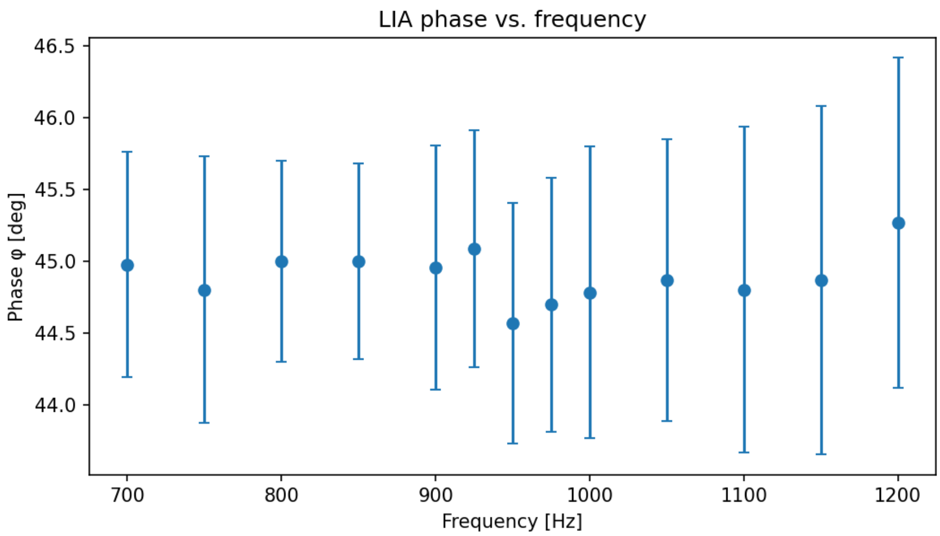

Figure 5: Measured phase $\varphi(f)$ of the lock-in amplifier output. The phase remains close to 45°.

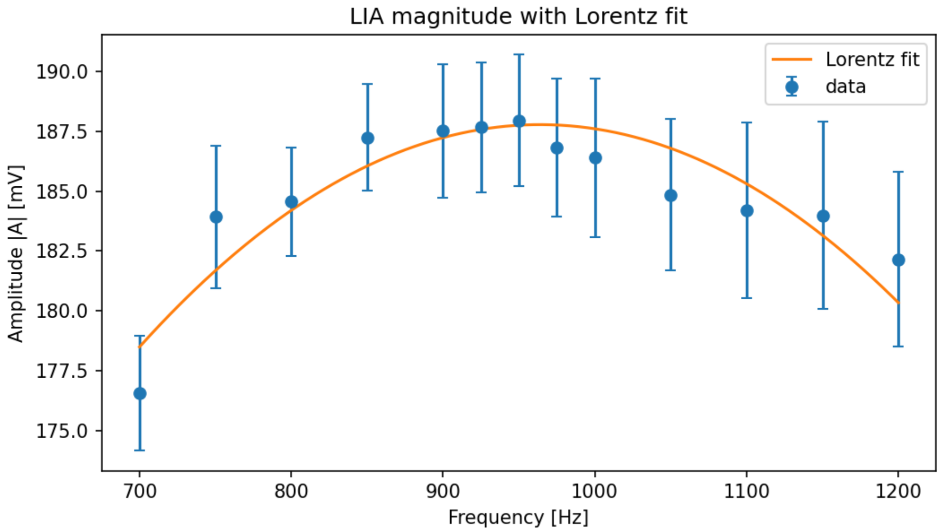

Figure 6: Lorentzian fit to the magnitude data. The fit yields an approximate resonance frequency f_0 and width Γ, from which the quality factor $Q = f_0/\Gamma$ can be estimated.

The peak point of the Lorentzian fit was extracted as

$$f_{\text{peak}} = 963.9 \pm 23.7\,\text{Hz}, \qquad A_{\text{peak}} = 187.8 \pm \text{nan}\,\text{mV}.$$

The peak frequency of the Lorentz fit can be identified with the resonance frequency of the oscillator,

$$f_0 = f_{\text{peak}}\,.$$

According to the Introduction, the linewidth in angular frequency is defined as

$$\Delta\omega = \Gamma,$$

and the quality factor is

$$Q = \frac{\omega_0}{\Gamma}, \qquad \text{with } \omega_0 = 2\pi f_0.$$

In our analysis the Lorentzian is fitted as a function of linear frequency f (in Hz). The fit returns a full width at half maximum (FWHM) directly in frequency units, which we denote by Δf. The corresponding angular linewidth is therefore

$$\Gamma = \Delta\omega = 2\pi\,\Delta f = 4.7 \times 10^4 \pm 4.9 \times 10^7\text{Hz},$$

so that

$$Q = \frac{\omega_0}{\Gamma} = \frac{2\pi f_0}{2\pi\Delta f} = \frac{f_0}{\Delta f}.$$

The Lorentzian fit function used in the lock-in amplifier analysis is

$$A(f) = C + \frac{A_0(\Gamma/2)^2}{(f - f_0)^2 + (\Gamma/2)^2},$$

where f_0 is the resonance frequency and the fit parameter Γ is the full width at half maximum (FWHM). With this parametrization,

$$\Delta f = \Gamma$$

Thus, once the peak frequency is obtained from the fit,

$$f_{\text{peak}} = f_0$$

the quality factor follows directly as

$$Q = \frac{f_0}{\Delta f} = \frac{f_0}{\Gamma} = 0.02 \pm 21.60.$$

Ringdown Measurement:

The voltage response of the mechanical resonator after a short excitation was analyzed in Python. The signal was detrended, a Fourier transform was used to estimate the resonant frequency f_0, and the amplitude envelope was fitted with an exponential decay

$$A(t) = A_0\, e^{-t/\tau},$$

from which the decay constant τ and the quality factor $Q = \pi f_0 \tau$ were obtained.

Table 2: Summary of extracted parameters from the ringdown data.

Parameter	**Symbol**	**Value ± Unc. (1σ)**	**Unit**
Sampling interval	Δt	0.001	s
Sampling rate	f_s	1000	Hz
Decay constant	τ	2713 ± 1713	s
Inverse decay rate	$1/\tau$	3.690e−4	s^{-1}
Resonant frequency	f_0	50.04 ± 0.03	Hz
Quality factor	Q	4.27e5 ± 2.69e5	–

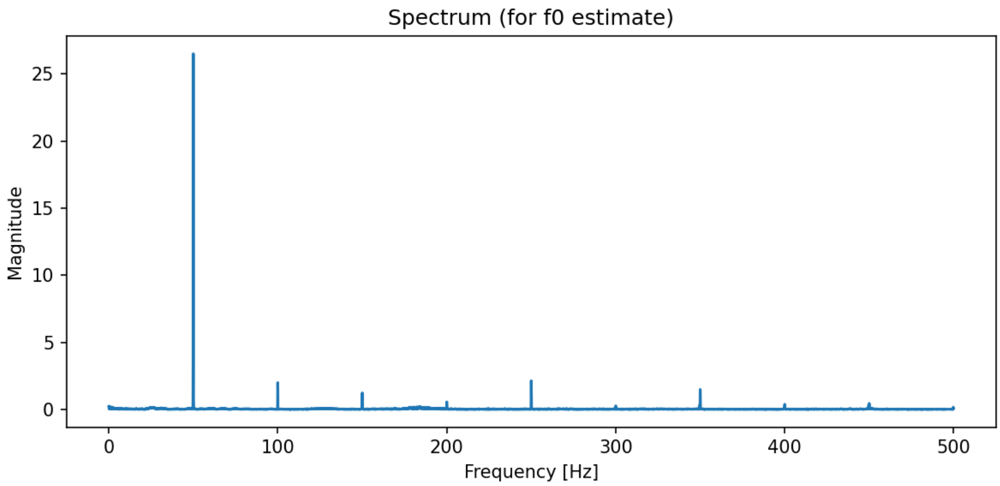

Figure 7: **Spectrum (for f_0 estimate).** Magnitude of the Fourier transform of the ringdown signal. The dominant peak near 50 Hz identifies the resonant frequency f_0.

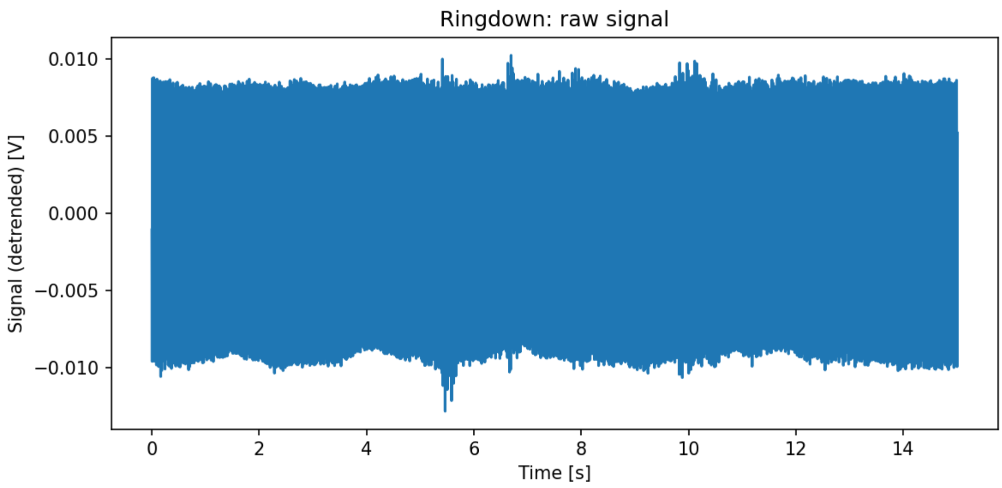

Figure 8: **Ringdown: raw signal.** Detrended time-domain voltage over the full acquisition window.

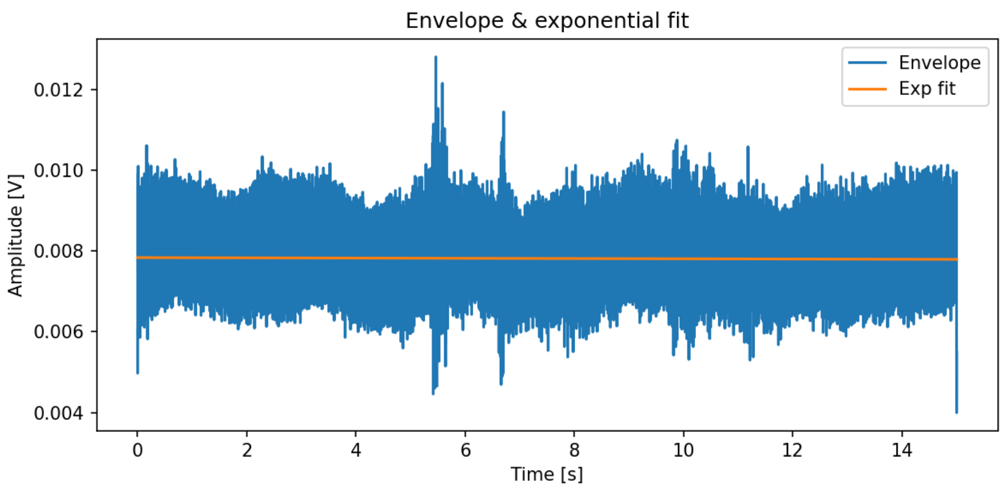

Figure 9: **Envelope and exponential fit.** Amplitude envelope (blue) and fitted exponential $A(t) = A_0 e^{-t/\tau}$ (orange), from which τ and Q are extracted.

Discussion

Lock-In Amplifier Measurement Although the resonance frequency is reasonably well determined ($f_0 \approx 964\,\mathrm{Hz}$), the Lorentzian amplitude profile measured with the lock-in amplifier is extremely flat. As a result, the width parameter returned by the fit is

$$\Gamma \approx 4.7 \times 10^4\,\mathrm{Hz}$$

with an uncertainty of

$$\Delta\Gamma \approx 4.9 \times 10^7\,\mathrm{Hz},$$

which exceeds the fitted value by several orders of magnitude. This leads to a formally computed quality factor of

$$Q = 0.02 \pm 21.60,$$

which is physically meaningless. This demonstrates that the frequency sweep performed with the lock-in amplifier does not contain enough curvature in the magnitude response to reliably extract the linewidth of the mechanical resonance. In other words, the resonance is too broad compared to the scanned frequency range, and the signal-to-noise ratio in the amplitude is insufficient to obtain a trustworthy estimate of Γ. Therefore, the lock-in amplitude sweep cannot be used to determine the quality factor, and only the resonance frequency f_0 can be extracted reliably from this measurement. A separate ringdown measurement is required to determine the damping rate and the true value of the quality factor. The phase response remained nearly constant around ϕ 44.9 ± 0.2 °, as expected for a nearly in-phase reference configuration.

Ringdown The resonator exhibits $f_0 = 50.04 \pm 0.03$ Hz and an exceptionally high quality factor $Q \approx 4 \times 10^5$, indicating very low damping and long energy storage. The relatively large uncertainty in τ is consistent with noise and the sensitivity of the envelope fit. This result contrasts with the lock-in measurement, where the much higher operating frequency and different excitation conditions lead to a lower apparent Q. The high Q observed here suggests that at low frequencies, the resonator behaves as a nearly ideal mechanical oscillator with minimal energy loss per cycle.

Comparison and Interpretation The two methods probe distinct regimes of the resonator's dynamics: the lock-in sweep measures the steady-state driven response in the kilohertz range, whereas the ringdown measures the free decay of a low-frequency mode near 50 Hz. Consequently, the observed Q values differ by several orders of magnitude. This discrepancy can be attributed to (i) different mechanical modes being excited, (ii) frequency-dependent loss mechanisms, (iii) varying readout sensitivities or (iv) measurement error and faulty measurement, for example through external vibrations when someone bumps in to the table. While the Lorentzian fit provides only qualitative information, the exponential ringdown fit directly quantifies the mechanical damping.

Uncertainties and Limitations Uncertainties for the lock-in data were propagated from δX and δY through Gaussian error propagation, while the ringdown fit uncertainty arises primarily from fluctuations in the exponential envelope due to noise. Because the amplitude–frequency curve was relatively flat, the Lorentzian fit parameters f_0 and Γ have large relative errors. In contrast, the ringdown measurement shows a well-defined decay constant τ, although affected by baseline drift and quantization noise.

Conclusion

In this experiment, the dynamic behavior of a mechanical resonator was investigated using two complementary approaches: frequency-domain measurements with a lock-in amplifier and time-domain analysis through a ringdown experiment. The lock-in measurement provided the amplitude and phase response over a range of driving frequencies, revealing a broad resonance at 963.9 ± 23.7 Hz and a relatively low quality factor of $Q = 0.0206 \pm 21.6$. In contrast, the ringdown analysis yielded a resonance at $f_0 = 50.04 \pm 0.03$ Hz with a very high quality factor of $Q \approx 4 \times 10^5$, indicating extremely low damping and efficient energy storage.

The comparison between the two methods demonstrates how measurement technique and frequency range influence the observed dynamics: while the driven frequency sweep provides qualitative insight into resonance behavior, the ringdown method directly quantifies intrinsic energy losses. The combination of both approaches offers a comprehensive understanding of the resonator's properties across different regimes. Such mechanical resonators, characterized by high Q and low damping, are fundamental in precision sensing, vibration isolation, and frequency standard applications.

Appendix

AI Usage Declaration

This report was prepared with assistance from ChatGPT (OpenAI) for language polishing, formatting suggestions, and for helping to develop, debug, and correct the syntax of the Python code. The AI was not used to generate raw data, to select results, or to determine the final conclusions. All scientific content (methods, calculations, results, and interpretation) was critically reviewed and validated by the authors. The authors take full responsibility for the accuracy of the analysis and for any remaining errors. No confidential or personal data were provided to the AI system during the preparation of this report.

Lock-in calculation results

Table 3: Lock-in amplitude R and phase θ with propagated uncertainties.

f / Hz	$R \pm \Delta R$ / V	$\theta \pm \Delta\theta$ / rad
700	0.1766 ± 0.0024	0.7850 ± 0.0137
750	0.1839 ± 0.0030	0.7819 ± 0.0162
800	0.1846 ± 0.0023	0.7854 ± 0.0122
850	0.1872 ± 0.0022	0.7854 ± 0.0119
900	0.1875 ± 0.0028	0.7846 ± 0.0149
925	0.1877 ± 0.0027	0.7869 ± 0.0144
950	0.1880 ± 0.0028	0.7779 ± 0.0146
975	0.1868 ± 0.0029	0.7801 ± 0.0154
1000	0.1864 ± 0.0033	0.7816 ± 0.0178
1050	0.1848 ± 0.0032	0.7831 ± 0.0171
1100	0.1842 ± 0.0037	0.7819 ± 0.0198
1150	0.1840 ± 0.0039	0.7831 ± 0.0212
1200	0.1822 ± 0.0037	0.7901 ± 0.0201

References

[1] *Physikpraktikum für 3. Semester — Experiment Mechanischer Resonator 1.* Lab manual. Restricted access (ETH login required). ETH Zürich, Departement Physik, Sept. 26, 2024. URL: https://ap.phys.ethz.ch/Anleitungen/Bilingual/XX-MechResonator.pdf.

Python Code Lock-In Amplifier Measurement

```
import numpy as np
import matplotlib.pyplot as plt
from math import atan2, sqrt
from dataclasses import dataclass

# ---------- 0) Messdaten aus der Tabelle (mV, Hz) ----------
# f in Hz, X, dX, Y, dY in mV (aus deinem Screenshot abgetippt)
f   = np.array([700,750,800,850,900,925,950,975,1000,1050,1100,1150,1200], dtype=float)
X   = np.array([124.9,130.5,130.5,132.4,132.7,132.5,133.9,132.8,132.3,131.0,130.7,130.4,128.2])
dX  = np.array([  1.8,   3.4,   2.7,   2.5,   3.2,   2.9,   2.8,   3.3,   3.6,   3.4,   3.8,   4.1,   3.9])
Y   = np.array([124.8,129.6,130.5,132.4,132.5,132.9,131.9,131.4,131.3,130.4,129.8,129.8,129.4])
dY  = np.array([  2.9,   2.5,   1.7,   1.9,   2.3,   2.5,   2.7,   2.4,   3.0,   2.9,   3.5,   3.7,   3.4])

# Einheit auf V umrechnen
Xv, dXv, Yv, dYv = X*1e-3, dX*1e-3, Y*1e-3, dY*1e-3

# ---------- 1) Betrag & Phase + Unsicherheiten ----------
A = np.sqrt(Xv**2 + Yv**2)                         # Betrag [V]
phi = np.arctan2(Yv, Xv)                           # Phase [rad]

# Fehlerfortpflanzung (unabhngige X,Y):
# A = sqrt(X^2+Y^2)  -> u_A^2 = (X/A)^2 u_X^2 + (Y/A)^2 u_Y^2
uA = np.sqrt( (Xv/A)**2 * dXv**2 + (Yv/A)**2 * dYv**2 )

# phi = atan2(Y,X) -> u_phi^2  (/X)^2 u_X^2 + (/Y)^2 u_Y^2
# /X = -Y/(X^2+Y^2) , /Y =  X/(X^2+Y^2)
den = (Xv**2 + Yv**2)
uphi = np.sqrt( ( (-Yv/den)**2 ) * dXv**2 + ( (Xv/den)**2 ) * dYv**2 )

# ---------- 2) Plots (mit Fehlerbalken) ----------
plt.figure(figsize=(7.2,4.2))
plt.errorbar(f, A*1e3, yerr=uA*1e3, fmt='o', capsize=3)
plt.xlabel("Frequency [Hz]")
plt.ylabel("Amplitude |A| [mV]")
plt.title("LIA magnitude vs. frequency")
```

```
plt.tight_layout()
plt.savefig("lia_magnitude_vs_frequency.png", dpi=150)
plt.close()

plt.figure(figsize=(7.2,4.2))
plt.errorbar(f, np.degrees(phi), yerr=np.degrees(uphi), fmt='o', capsize=3)
plt.xlabel("Frequency [Hz]")
plt.ylabel("Phase  [deg]")
plt.title("LIA phase vs. frequency")
plt.tight_layout()
plt.savefig("lia_phase_vs_frequency.png", dpi=150)
plt.close()

# ---------- 3) (Optional) Lorentz-Fit fr A(f) ----------
# Die Daten sind relativ flach; wir versuchen trotzdem einen Lorentz auf den Betrag.
from scipy.optimize import curve_fit

def lorentz(f, A0, f0, gamma, C):
    # C = Basisoffset, gamma = FWHM
    return C + A0 * (0.5*gamma)**2 / ((f - f0)**2 + (0.5*gamma)**2)

try:
    p0 = [ (A.max()-A.min()), f[np.argmax(A)], 50.0, A.min() ]  # grobe Startwerte
    popt, pcov = curve_fit(lorentz, f, A, sigma=uA, absolute_sigma=True, p0=p0, maxfev=10000)
    A0_fit, f0_fit, gamma_fit, C_fit = popt
    perr = np.sqrt(np.diag(pcov))
    A0_err, f0_err, gamma_err, C_err = perr
    Q_fit = f0_fit/gamma_fit if gamma_fit>0 else np.nan

    # ---------- Peak mit Unsicherheiten ----------
    # Maximum liegt bei f0
    f_peak = f0_fit
    u_f_peak = f0_err

    # Amplitude am Peak: A_peak = C + A0
    A_peak = C_fit + A0_fit

    # Fehler von A_peak: Var(C + A0) = Var(C) + Var(A0) + 2 Cov(C, A0)
```

```
    var_A_peak = pcov[0,0] + pcov[3,3] + 2*pcov[0,3]
    u_A_peak = np.sqrt(var_A_peak) if var_A_peak > 0 else np.nan

    print("\nLorentz peak (from fit):")
    print(f"  f_peak  = {f_peak:.3f} +/- {u_f_peak:.3f} Hz")
    print(f"  A_peak  = {A_peak*1e3:.3f} +/- {u_A_peak*1e3:.3f} mV")

    # ---------- Plot mit Fit ----------
    ff = np.linspace(f.min(), f.max(), 500)
    plt.figure(figsize=(7.2,4.2))
    plt.errorbar(f, A*1e3, yerr=uA*1e3, fmt='o', capsize=3, label="data")
    plt.plot(ff, lorentz(ff, *popt)*1e3, label="Lorentz fit")
    plt.xlabel("Frequency [Hz]"); plt.ylabel("Amplitude |A| [mV]")
    plt.title("LIA magnitude with Lorentz fit")
    plt.legend()
    plt.tight_layout()
    plt.savefig("lia_magnitude_fit.png", dpi=150)
    plt.close()
except Exception as e:
    A0_fit=f0_fit=gamma_fit=C_fit=Q_fit=np.nan
    A0_err=f0_err=gamma_err=C_err=np.nan


# ---------- 4) OS vs LIA Vergleich aus dem Results-Absatz ----------
@dataclass
class Meas:
    val: float
    u: float

def ratio_with_u(a: Meas, b: Meas):
    r = a.val/b.val
    ur = r * np.sqrt( (a.u/a.val)**2 + (b.u/b.val)**2 )
    return r, ur

# (Passe diese Zahlen an, falls in deinem Text andere stehen)
f_OS  = Meas(498.7, 0.2)         # Hz
f_LIA = Meas(500.7, 0.0001)      # Hz
V_OS  = Meas(9.975, 0.700)       # V
V_LIA = Meas(9.972, 0.003)       # V

R, uR = ratio_with_u(V_OS, V_LIA)
```

```
df    = f_OS.val - f_LIA.val
udf   = np.sqrt(f_OS.u**2 + f_LIA.u**2)

# ---------- 5) Report ----------
print("\n=== LIA table analysis ===")
print(f"N points: {len(f)}")
print(f"|A| range: {A.min()*1e3:.2f} .. {A.max()*1e3:.2f} mV")
print(f"Phase range: {np.degrees(phi).min():.2f} .. {np.degrees(phi).max():.2f}
    deg")

print("\nLorentz fit on |A|(f):")
print(f"  f0    = {f0_fit:.3f} +/- {f0_err if np.isfinite(f0_err) else
    np.nan:.3f} Hz")
print(f"  Gamma= {gamma_fit:.3f} +/- {gamma_err if np.isfinite(gamma_err) else
    np.nan:.3f} Hz   (FWHM)")

# Q und Fehler von Q
if gamma_fit > 0 and np.isfinite(f0_err) and np.isfinite(gamma_err):
    Q_fit = f0_fit / gamma_fit
    uQ = Q_fit * np.sqrt( (f0_err/f0_fit)**2 + (gamma_err/gamma_fit)**2 )
else:
    Q_fit = np.nan
    uQ = np.nan

print(f"  Q     = {Q_fit:.3g} +/- {uQ:.3g}")
print("  (If values are NaN or uQ >> Q, the curve is too flat for a meaningful
    resonance fit.)")

print("\nOS vs LIA (from Results text):")
print(f"  df = f_OS - f_LIA = {df:.3f} +/- {udf:.3f} Hz")
print(f"  Voltage ratio R = Vpp_OS / Vpp_LIA = {R:.4f} +/- {uR:.4f}")

print("\nSaved figures:")
print("  lia_magnitude_vs_frequency.png")
print("  lia_phase_vs_frequency.png")
print("  lia_magnitude_fit.png  (only if fit succeeded)")

# ---------- 6) Mittelwert der Phase mit Unsicherheit ----------
phi_deg = np.degrees(phi)
uphi_deg = np.degrees(uphi)

```

```
# gewichteter Mittelwert mit Unsicherheiten
w = 1 / uphi_deg**2
phi_mean = np.sum(w * phi_deg) / np.sum(w)
u_phi_mean = np.sqrt(1 / np.sum(w))

print("\nWeighted mean phase:")
print(f"  phi_mean = {phi_mean:.3f} +/- {u_phi_mean:.3f} deg")




# ---------- 7) Tabelle fr R  dR und theta  dtheta (rad) ----------

# R ist einfach die Amplitude A (in V), theta die Phase phi (in rad)
R = A           # [V]
dR = uA         # [V]
theta = phi     # [rad]
dtheta = uphi   # [rad]

print("\nTable: f, R  dR [V], theta  dtheta [rad]")
print(f"{'f [Hz]':>6}  {'R [V]':>10}  {'dR [V]':>10}  {'theta [rad]':>12}  {'dtheta [rad]':>12}")
for fi, Ri, dRi, ti, dti in zip(f, R, dR, theta, dtheta):
    print(f"{fi:6.0f}  {Ri:10.6f}  {dRi:10.6f}  {ti:12.6f}  {dti:12.6f}")
```

Python Code Ringdown Measurement

```python
# ringdown_analysis.py
# Minimal script: load ringdown CSV, fit exponential decay, estimate f0 & Q, save plots + print summary.

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from pathlib import Path

CSV_PATH = "ringdown.csv"   # <- Datei hierher legen oder Pfad anpassen

# --- 1) CSV laden (mit Logger-Kommentarzeilen) ---
# erkennt Trennzeichen automatisch; berspringt Zeilen, die mit '%' beginnen; keine Kopfzeile vorhanden
df = pd.read_csv(CSV_PATH, comment="%", sep=None, engine="python", header=None, names=["t","x"])
t = df["t"].to_numpy(dtype=float)
x_raw = df["x"].to_numpy(dtype=float)

# --- 2) Vorverarbeitung ---
# versuche SciPy fr sauberes Detrend/Hilbert, sonst einfache Alternativen
try:
    from scipy.signal import hilbert, detrend
    SCIPY = True
except Exception:
    SCIPY = False

x = (x_raw - np.mean(x_raw)) if not SCIPY else detrend(x_raw, type="constant")

dt = np.median(np.diff(t))
fs = 1.0 / dt if dt > 0 else np.nan

# --- 3) Hllkurve (Envelope) ---
if SCIPY:
    env = np.abs(hilbert(x))
else:
    # einfache Hllkurve: |x| gltten
    absx = np.abs(x)
    win = max(5, int(len(absx) * 0.005))
    if win % 2 == 0: win += 1
```

```
    ker = np.ones(win) / win
    env = np.convolve(absx, ker, mode="same")

# --- 4) Exponential-Fit: env ~ A0 * exp(-t / tau) ---
n = len(t)
i0, i1 = int(0.02*n), int(0.90*n)    # Fit nur in ruhiger Mitte
env_clip = np.clip(env, np.max(env)*1e-6, None)
fit_t = t[i0:i1]
fit_env = env_clip[i0:i1]

y = np.log(fit_env)
A = np.vstack([np.ones_like(fit_t), -fit_t]).T
coef, _, _, _ = np.linalg.lstsq(A, y, rcond=None)
lnA0, neg_inv_tau = coef
tau = 1.0 / neg_inv_tau
A0 = np.exp(lnA0)

# Unsicherheit grob aus Residuen der Log-Fits
y_fit = lnA0 - fit_t / tau
res = y - y_fit
sigma = np.std(res)
Sxx = np.sum((fit_t - np.mean(fit_t))**2)
std_slope = sigma / np.sqrt(Sxx) if Sxx > 0 else np.nan
tau_std = (tau**2) * std_slope if np.isfinite(std_slope) else np.nan


# --- 5) Frequenzschtzung f0 via FFT ---
xw = x * np.hanning(n)  # Fensterung, damit die FFT hbscher aussieht
freqs = np.fft.rfftfreq(n, d=dt) if dt > 0 else np.arange(len(np.fft.rfft(xw)))
Xf = np.fft.rfft(xw)
mag = np.abs(Xf)
peak_idx = np.argmax(mag[1:]) + 1 if len(mag) > 1 else 0
f0 = float(freqs[peak_idx]) if np.ndim(freqs) else float(peak_idx)

# Frequenzauflsung und Fehler von f0
if len(freqs) > 1:
    df_fft = float(freqs[1] - freqs[0])   # Binbreite
    f0_std = 0.5 * df_fft                 # einfache 1-Abschtzung: halbe Binbreite
else:
    df_fft = np.nan
```

```
    f0_std = np.nan



# --- 6) Qualittsfaktor Q ---
Q = np.pi * f0 * tau if np.isfinite(f0) else np.nan
Q_std = Q * (tau_std / tau) if (np.isfinite(Q) and np.isfinite(tau_std)) else
    np.nan

# --- 7) Plots speichern ---
out_dir = Path(".")
# Rohsignal
plt.figure(figsize=(8,4))
plt.plot(t, x)
plt.xlabel("Time [s]"); plt.ylabel("Signal (detrended) [V]")
plt.title("Ringdown: raw signal")
plt.tight_layout()
plt.savefig(out_dir/"ringdown_raw.png", dpi=150)
plt.close()

# Envelope + Fit
plt.figure(figsize=(8,4))
plt.plot(t, env, label="Envelope")
plt.plot(t, A0*np.exp(-t/tau), label="Exp fit")
plt.xlabel("Time [s]"); plt.ylabel("Amplitude [V]")
plt.title("Envelope & exponential fit")
plt.legend()
plt.tight_layout()
plt.savefig(out_dir/"ringdown_envelope_fit.png", dpi=150)
plt.close()

# Spektrum
plt.figure(figsize=(8,4))
plt.plot(freqs, mag)
plt.xlabel("Frequency [Hz]"); plt.ylabel("Magnitude")
plt.title("Spectrum (for f0 estimate)")
plt.tight_layout()
plt.savefig(out_dir/"ringdown_spectrum.png", dpi=150)
plt.close()

# --- 8) Ergebnisse ausgeben + speichern ---
```

```
print("\n=== Ringdown Analysis Summary ===")
print(f"Samples: {n}")
print(f"dt: {dt:.9g} s   fs: {fs:.9g} Hz")
print(f"Tau: {tau:.9g} s   {tau_std if np.isfinite(tau_std) else np.nan:.3g} s")
print(f"Decay rate 1/tau: {1.0/tau:.9g} 1/s")
print(f"f0 (FFT peak): {f0:.9g} Hz   {f0_std if np.isfinite(f0_std) else np.nan:.3g} Hz")
print(f"Q: {Q:.9g}   {Q_std if np.isfinite(Q_std) else np.nan:.3g}")
print("Saved: ringdown_raw.png, ringdown_envelope_fit.png, ringdown_spectrum.png")

# CSV mit Kennwerten
pd.DataFrame({
    "Metric": ["dt","fs","tau","1/tau","f0","Q"],
    "Value":  [dt, fs, tau, 1.0/tau, f0, Q],
    "Uncertainty(1)": [np.nan, np.nan, tau_std, (tau_std/(tau**2)) if (np.isfinite(tau) and np.isfinite(tau_std)) else np.nan, np.nan, Q_std]
}).to_csv(out_dir/"ringdown_results.csv", index=False)
print("Saved: ringdown_results.csv")
```

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Mechanical Resonator 1

Homework

- ☒ I understand the concept of the complex plane and how it relates to the current experiment.
- ☒ I understand the concept of a Fourier Transform and how it relates to the current experiment.
- ☒ I have installed and tested that the software runs on my computer.
- ☒ I have read the "Getting started"manual and tried out a few things with the Moku:Go simulator in the software.
- ☒ I know what dB is and how to handle them:

dB expresses ratio quantities on logarithmic scale: for power $L_{DB} = 10 \log_{10}(P_2/P_1)$, for voltage amplitude $A_{dB} = 20 \log_{10}(V_2/V_1)$

1 Comparison

Oscilloscope (OS)

Measured quantities

V_{meas} = 9.975 ± 0.700 V (amplitude)

f_{meas} = 498.7 Hz ± 0.2 Hz (frequency)

Advantages	Disadvantages
+ Direct time-domain view of signals	- Limited sensitivity – small signals get buried in noise
+ High bandwidth – shows waveform shape, noise	- No selective noise or frequency surpression
+ Intuitive: Let's you see what happens	- Not suitable for precise amplitude/phase measurement of very weak signals

Lock-in Amplifier (LIA)

Tick and note all relevant settings for a meaningful measurement:

☐ Phase shift $\Delta\phi$ = ____________ ☐ Gain G = ____________

☐ Lowpass filter f_{filter} = ____________

Measured quantities

V_{meas} = 9.972 V ± 300 mV (amplitude)

f_{meas} = 500.7 Hz ± 100 μHz (frequency)

Advantages	Disadvantages
+ Extremely sensitive: detects smaller signals buried in noise	- Requires a reference signal (measures only at one frequency)
+ Phase sensitivity detection gives both amplitude & phase difference	- Slow response (filter time constants) — can not capture fast transients
+ Good noise rejection & dynamic range	- More complex setup: strong out-of-bands signals may cause distortion

(OS) V_{pp} = 1.003 ± 0.0702 R (LIA)

2 Resonance with LIA

Magnitude · calculate · Phase Shift

	f (Hz)	A: $X \pm \delta X$ (V)	B: $Y \pm \delta Y$ (V)	$R \pm \delta R$ (V)	~~$\theta \pm \delta\theta$ (V)~~	$\theta \pm \delta\theta$ (rad)
1	700	124.9 ± 1.8	124.8 ± 2.9	0.1766 ± 0.0024		0.7850 ± 0.0137
2	750	130.5 ± 3.4	129.6 ± 2.5	0.1839 ± 0.0030		0.7819 ± 0.0162
3	800	130.5 ± 2.7	130.5 ± 1.7	0.1846 ± 0.0023		0.7854 ± 0.0122
4	850	132.4 ± 2.5	132.4 ± 1.9	0.1872 ± 0.0022		0.7854 ± 0.0119
5	900	132.7 ± 3.2	132.5 ± 2.3	0.1875 ± 0.0028		0.7846 ± 0.0149
6	925	132.5 ± 2.9	132.9 ± 2.5	0.1877 ± 0.0027		0.7869 ± 0.0144
7	950	133.9 ± 2.8	131.9 ± 2.7	0.1880 ± 0.0028		0.7779 ± 0.0146
8	975	132.8 ± 3.3	131.4 ± 2.4	0.1868 ± 0.0029		0.7801 ± 0.0154
9	1'000	132.3 ± 3.6	131.3 ± 3.0	0.1864 ± 0.0033		0.7816 ± 0.0178
10	1'050	131.0 ± 3.4	130.4 ± 2.9	0.1848 ± 0.0032		0.7831 ± 0.0171
11	1'100	130.7 ± 3.8	129.8 ± 3.5	0.1842 ± 0.0037		0.7819 ± 0.0198
12	1'150	130.4 ± 4.1	129.8 ± 3.7	0.1840 ± 0.0039		0.7831 ± 0.0212
13	1200	128.2 ± 3.9	129.4 ± 3.4	0.1822 ± 0.0037		0.7904 ± 0.0201

3 Resonance with FRA

☒ Measurements are taken in the correct units.

☒ I found the resonance and have the correct filter settings:

Given { Averaging: 2.00 ms settling time: 100ms ms

☒ I have stored at least two datasets on my computer to plot at home.

4 Ringdown

- I have used the OS / LIA to take the measurement (pick one).

☒ I have stored at least two datasets on my computer to plot at home.

☒ I have checked that the data I saved has sufficient points to plot and fit a model to them.