Piezoelectric Effect

Physics Laboratory 3

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Abstract In this experiment, the direct piezoelectric effect was investigated for quartz and barium titanate. An oscillating mechanical force was applied to the samples using a spring-lever setup, and the generated voltage was measured with an oscilloscope. By plotting the voltage as a function of the lever factor, the piezoelectric coefficient d_{11} was determined from the slope of a linear fit. For quartz, the coefficient was found to be nearly temperature-independent, with an average value of $\bar{d}_{11} = (2.87 \pm 0.10) \times 10^{-12}\ \mathrm{C\,N^{-1}}$, in reasonable agreement with the literature value. Barium titanate showed a much stronger dependence on temperature and polarization state. The polarized sample produced significantly larger piezoelectric coefficients than the unpolarized sample, while the response above the Curie temperature was strongly reduced and close to zero. These results confirm the stable piezoelectric behavior of quartz and demonstrate the importance of ferroelectric domain alignment for the macroscopic piezoelectric response of barium titanate.

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1 Introduction

Piezoelectricity describes the coupling between mechanical and electrical properties in certain crystalline materials. If mechanical stress is applied to a non-centrosymmetric crystal, an electric polarization is generated and surface charges appear. This phenomenon is known as the direct piezoelectric effect [3]. Conversely, an applied electric field can induce a mechanical deformation of the crystal, which is referred to as the inverse piezoelectric effect. In the regime of small deformations, these relations can be treated as linear, which makes piezoelectric materials particularly suitable for quantitative measurements.

In this experiment, the piezoelectric behavior of two different materials, quartz (SiO_2) and barium titanate ($BaTiO_3$), is investigated. Quartz is a classical piezoelectric material with comparatively stable properties and only a weak temperature dependence of its piezoelectric coefficient. For this reason, it is well suited as a reference material and is widely used in force sensors and frequency-stable oscillators. In contrast, barium titanate is a ferroelectric ceramic whose piezoelectric properties depend strongly on temperature and on the polarization state of the sample. Below its phase transition near 120 °C, barium titanate is in a ferroelectric phase and exhibits piezoelectric behavior, whereas above this transition the effect disappears.

The aim of this experiment is to examine the linear relation between the applied mechanical excitation and the generated voltage, to determine the piezoelectric coefficient d_{11} from the slope of this relation, and to compare the temperature dependence of the piezoelectric response of quartz and barium titanate [5]. In particular, polarized and unpolarized barium titanate samples are studied in order to analyze the influence of ferroelectric domain alignment on the measured piezoelectric coefficient. The experiment therefore combines a verification of the basic theoretical relations of piezoelectricity with an investigation of material-specific behavior and experimental uncertainties.

2 Experimental

The experimental setup consisted of a piezoelectric sample placed between two copper blocks inside a temperature-controlled oven. The sample temperature was monitored using two digital thermometers connected to temperature probes in the oven setup. Mechanical stress was applied to the sample by a spring-loaded lever system. By changing the lever setting x, the lever factor and therefore the effective force acting on the sample could be varied in a controlled way. For the measurements on barium titanate, a high-voltage source was additionally used to polarize or depolarize the sample.

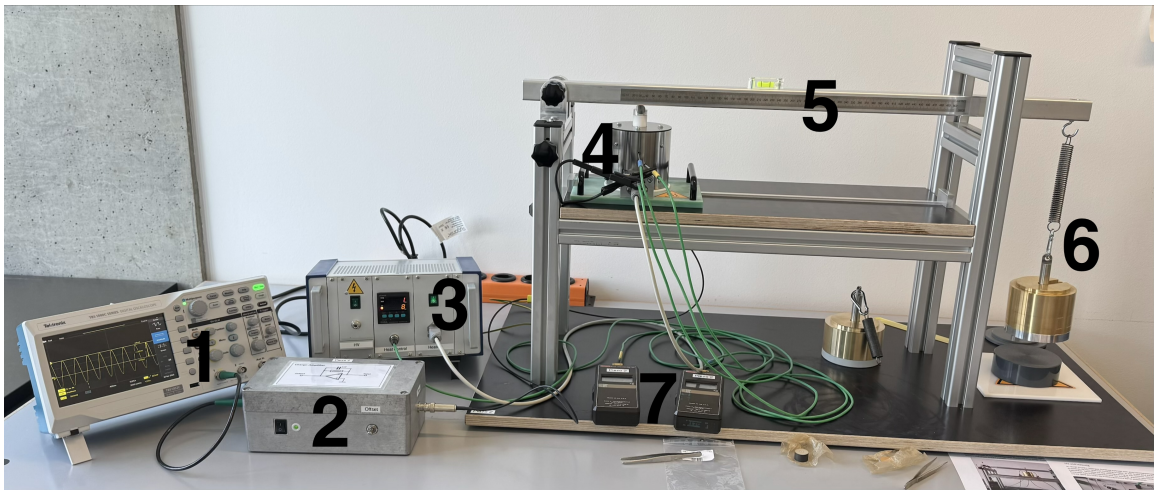


Figure 1: Experimental setup used to measure the direct piezoelectric effect. The main components are: (1) oscilloscope, (2) signal amplifier, (3) temperature and high-voltage control unit, (4) oven with sample holder, (5) spring-lever system, (6) oscillating spring with attached mass and (7) digital thermometers

Before each measurement, the crystals were cleaned with alcohol and were not touched with bare hands in order to avoid contamination of the surfaces. This was important because the experiment is sensitive to surface charge. The sample was placed flat between the copper blocks to ensure good mechanical contact and to prevent damage.

The spring was displaced by a fixed release elongation and then released. The attached mass was allowed to oscillate freely for 20 oscillations before the oscilloscope trace was frozen. The voltage amplitude U generated by the piezoelectric sample was then read from the oscilloscope. The oscillation period was determined from the time difference between two successive maxima of the signal, which was used to calculate the angular frequency ω.

For each lever setting, the lever factor f_{lever} was calculated from the geometry of the setup. Together with the calibrated spring constant and the fixed release elongation, this determined the force amplitude acting on the sample. The measured voltage U was then plotted as a function of f_{lever}. From the slope of a linear fit, the piezoelectric coefficient d_{11} was determined.

For the quartz crystal, measurements were performed at several temperatures in order to test the temperature dependence of its piezoelectric response. For barium titanate, measurements were carried out for both a polarized and an unpolarized sample during heating and cooling. In addition, a measurement above the Curie temperature was performed to investigate the suppression of the piezoelectric response in the paraelectric phase.

During the experiment, the setup was already properly grounded. The main safety precaution was to avoid direct contact with the heating oven and heated metal parts, since temperatures above 100 °C were reached. The oven temperature was kept below the allowed maximum temperature.

3 Results & Data Analysis

3.1 Spring Calibration

In order to determine the spring constant, the length of the spring was measured for different applied masses. According to Hooke's law, the restoring force of the spring is proportional to its extension,

$$F = k_{\text{spring}}\,\Delta l, \tag{1}$$

where k is the spring constant and Δl is the elongation of the spring. The elongation is given by

$$\Delta l = l - l_0, \tag{2}$$

where l_0 is the unloaded spring length and l is the spring length under load.
The applied force was calculated from the attached mass m via

$$F = mg, \tag{3}$$

with $g = 9.81\,\mathrm{m\,s^{-2}}$. Thus, the spring constant for each measurement point is

$$k_{\text{spring}} = \frac{F}{\Delta l} = \frac{mg}{l - l_0}. \tag{4}$$

The unloaded spring length was measured to be

$$l_0 = 65\,\mathrm{mm}. \tag{5}$$

The mass of the spring itself was 47 g. In the present evaluation, this was neglected, since the unloaded length l_0 was measured for the freely hanging spring without any additional mass, such that the spring's own weight is already included in the reference length.

3.1.1 Raw and calculated data

Table 1: Raw and calculated data for the spring calibration.

Mass m (g)	Spring length l (mm)	Elongation Δl (mm)	Force F (N)	Spring constant k ($\mathrm{N\,m^{-1}}$)
472	67	2	4.63	2315
1000	69	4	9.81	2453
2003	89	24	19.6	819
4023	143	78	39.5	506

The mean spring constant was found to be

$$\bar{k}_{\text{spring}} = (1.52 \pm 0.50) \times 10^3 \text{ N m}^{-1}, \tag{6}$$

where the uncertainty denotes the standard error of the mean. Since all subsequent measurements were carried out using a mass of 4.023 kg, the spring constant corresponding to this mass was used in the following, namely

$$k_{\text{spring}} = 506 \text{ N m}^{-1}.$$

3.1.2 Determination of the force K_x

The force acting on the sample is given by

$$K_x = f_{\text{lever}} K_0, \tag{7}$$

where f_{lever} is the lever factor and K_0 is the force amplitude of the oscillating spring. The latter was calculated from the spring constant and the release elongation,

$$K_0 = k_{\text{spring}} \Delta l. \tag{8}$$

With

$$k_{\text{spring}} = 506 \text{ N m}^{-1} \qquad \text{and} \qquad \Delta l = 78 \text{ mm} = 0.078 \text{ m}, \tag{9}$$

this gives

$$K_0 = 39.5 \text{ N}. \tag{10}$$

Hence, for each lever setting, the force acting on the crystal was obtained from

$$K_x = f_{\text{lever}} \cdot K_0 = f_{\text{lever}} \cdot 39.5 \text{ N}. \tag{11}$$

3.1.3 Determination of the oscillation frequency

To determine the oscillation frequency of the spring-mass system, the attached mass was first allowed to oscillate 20 times. The oscilloscope trace was then frozen, and the period T was determined from the time difference between two successive maxima of the signal. The frequency was calculated using

$$f = \frac{1}{T}, \tag{12}$$

and the angular frequency was obtained from

$$\omega = 2\pi f. \tag{13}$$

For the present measurement, the period was found to be

$$T = 658\,\mathrm{ms}, \tag{14}$$

which gives

$$f = 1.52\,\mathrm{Hz} \tag{15}$$

and

$$\omega = 9.55\,\mathrm{rad\,s^{-1}}. \tag{16}$$

3.2 Temperature Dependence

For each measurement point, the lever factor was calculated from the lever setting x using

$$f_{\mathrm{lever}} = \frac{780}{100 + (x - x_0)}, \tag{17}$$

with

$$x_0 = 30\,\mathrm{mm}. \tag{18}$$

The corresponding force K_x acting on the sample was then calculated from the calibrated setup relation. After displacing the spring, the spring with the attached weight was allowed to oscillate 20 times, after which the voltage was measured using an oscilloscope.

3.2.1 Determination of the piezoelectric coefficient d_{11}

For the present setup, the measured voltage U is related to the lever factor f_{lever} by

$$U = \frac{2R\omega}{\pi}\, d_{11}\, K_x = \frac{2R\omega}{\pi}\, d_{11}\, K_0 f_{\mathrm{lever}}, \tag{19}$$

with

$$R = 45.5\,\mathrm{M\Omega}, \qquad K_0 = 39.5\,\mathrm{N}, \qquad \omega = 9.55\,\mathrm{rad\,s^{-1}}. \tag{20}$$

Thus, U is expected to depend linearly on f_{lever}. For each temperature, the measured voltage was therefore plotted as a function of f_{lever}, and a linear fit of the form

$$U = M\, f_{\mathrm{lever}} + b \tag{21}$$

was performed. From the slope M, the piezoelectric coefficient follows as

$$d_{11} = \frac{\pi}{2RK_0\omega} \cdot M \tag{22}$$

Using the experimental constants above, this becomes

$$d_{11} \approx 9.15 \times 10^{-11} \, \frac{\mathrm{C}}{\mathrm{N\,V}} \cdot M, \tag{23}$$

where the slope M has to be inserted in volts.

3.3 Raw and calculated data

The complete raw and calculated data for all quartz and barium titanate measurements are given in the Appendix. The following analysis therefore focuses on the fitted slopes, the extracted piezoelectric coefficients, and their temperature dependence.

4 Discussion

4.1 Quartz Crystal

Figure 2 shows the measured voltage U as a function of the lever factor f_{lever} for the quartz crystal at different temperatures. For all temperatures, the data show an approximately linear dependence, as expected from

$$U = \frac{2RK_0\omega}{\pi} d_{11} f_{\text{lever}}. \tag{24}$$

This confirms that the measured voltage is proportional to the applied mechanical excitation, in agreement with the direct piezoelectric effect.

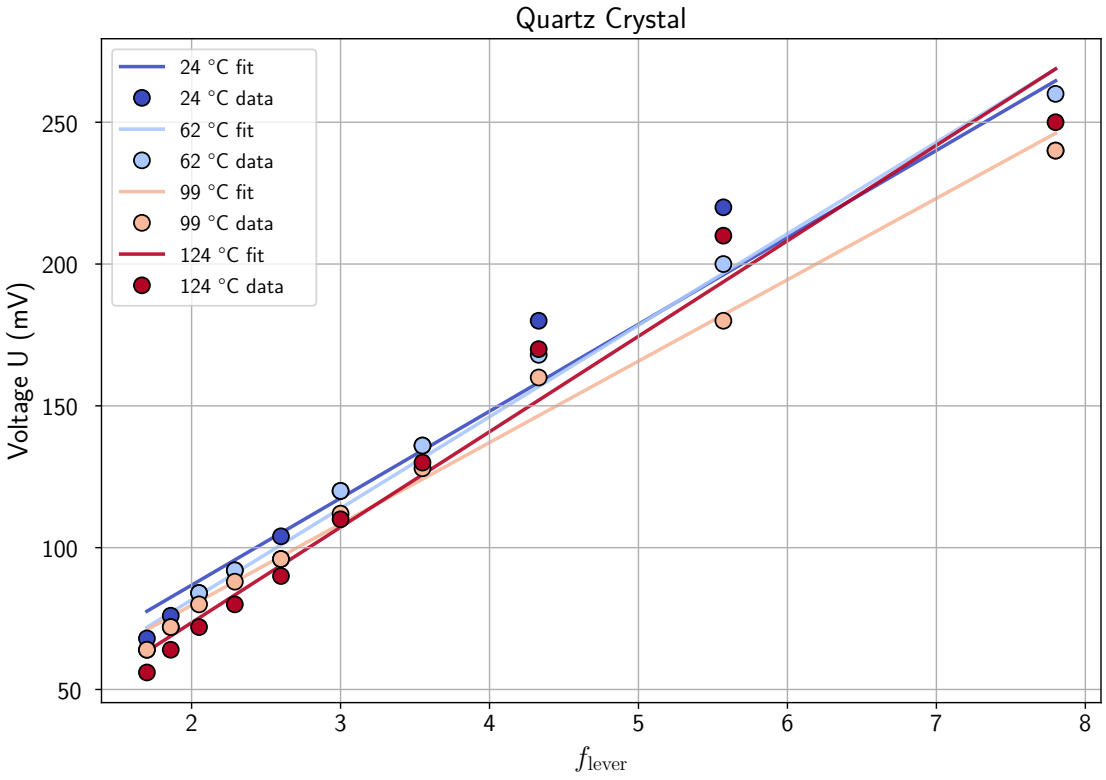


Figure 2: Measured voltage U as a function of the lever factor f_{lever} for the quartz crystal at different temperatures. The solid lines represent linear fits to the data.

From the slopes of the linear fits, the following piezoelectric coefficients were obtained:

$$d_{11}(24°\text{C}) = 2.81 \times 10^{-12}\ \text{C}\,\text{N}^{-1}, \tag{25}$$

$$d_{11}(62°\text{C}) = 2.95 \times 10^{-12}\ \text{C}\,\text{N}^{-1}, \tag{26}$$

$$d_{11}(99°\text{C}) = 2.63 \times 10^{-12}\ \text{C}\,\text{N}^{-1}, \tag{27}$$

$$d_{11}(124°\text{C}) = 3.08 \times 10^{-12}\ \text{C}\,\text{N}^{-1}. \tag{28}$$

Thus, the measured values lie in the range

$$d_{11} = (2.63\text{–}3.08) \times 10^{-12}\ \mathrm{C\,N^{-1}}. \tag{29}$$

The average value over all measured temperatures is

$$\bar{d}_{11} = (2.87 \pm 0.10) \times 10^{-12}\ \mathrm{C\,N^{-1}}, \tag{30}$$

where the uncertainty denotes the standard error of the mean.
The fitted values differ only moderately, which is consistent with the expectation that quartz has a weak temperature dependence of its piezoelectric coefficient. The obtained average value is in reasonable agreement with the literature value for quartz,

$$|d_{11,\mathrm{lit}}| = 2.3 \times 10^{-12}\ \mathrm{C\,N^{-1}}[1]. \tag{31}$$

Here, only the magnitude of the literature value is compared, since the sign depends on the chosen crystal-axis convention. The measured value is about 25% larger than the literature value, but it remains of the same order of magnitude. This deviation can be attributed to systematic uncertainties in the force calibration, finite oscilloscope reading accuracy, damping of the oscillating spring system, scatter of the data points, and small deviations from an ideal linear response. Overall, the quartz measurements show a comparatively stable piezoelectric response over the investigated temperature range, making quartz suitable as a reference material in this experiment.

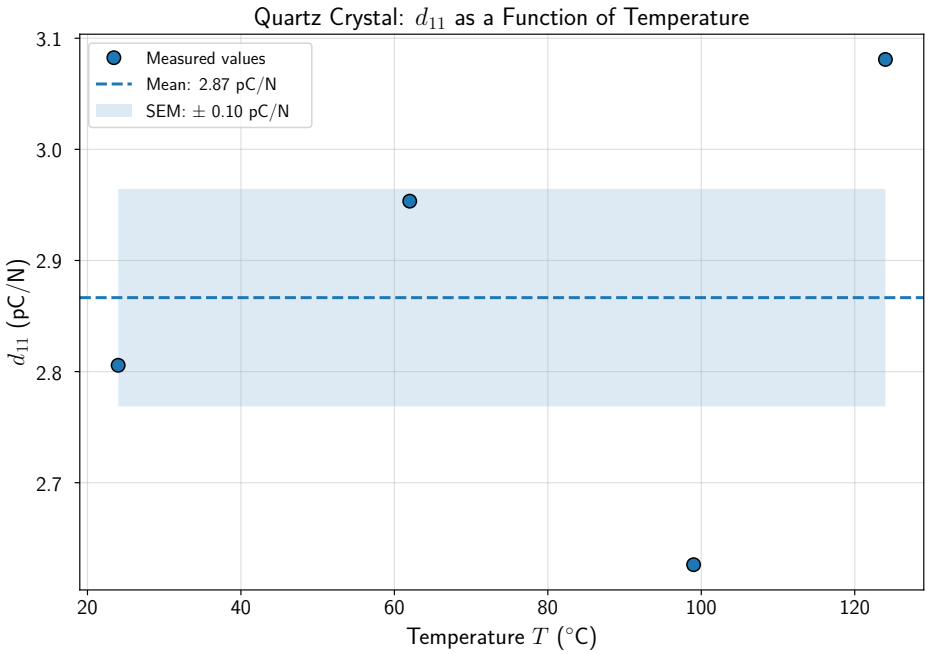


Figure 3: Piezoelectric coefficient d_{11} of quartz as a function of temperature. The dashed line shows the mean value, and the shaded region indicates the standard error of the mean.

Figure 3 summarizes the temperature dependence of the extracted piezoelectric coefficient for quartz. Within the experimental scatter, no systematic temperature trend is visible. This supports the conclusion that the piezoelectric response of quartz is only weakly temperature dependent in the investigated temperature range.

4.2 Barium Titanate above the Curie Temperature

Above the Curie temperature, barium titanate is expected to transform from the ferroelectric tetragonal phase into the paraelectric cubic phase [4]. Since the cubic phase is centrosymmetric, the direct piezoelectric effect should vanish or at least become very small [5].

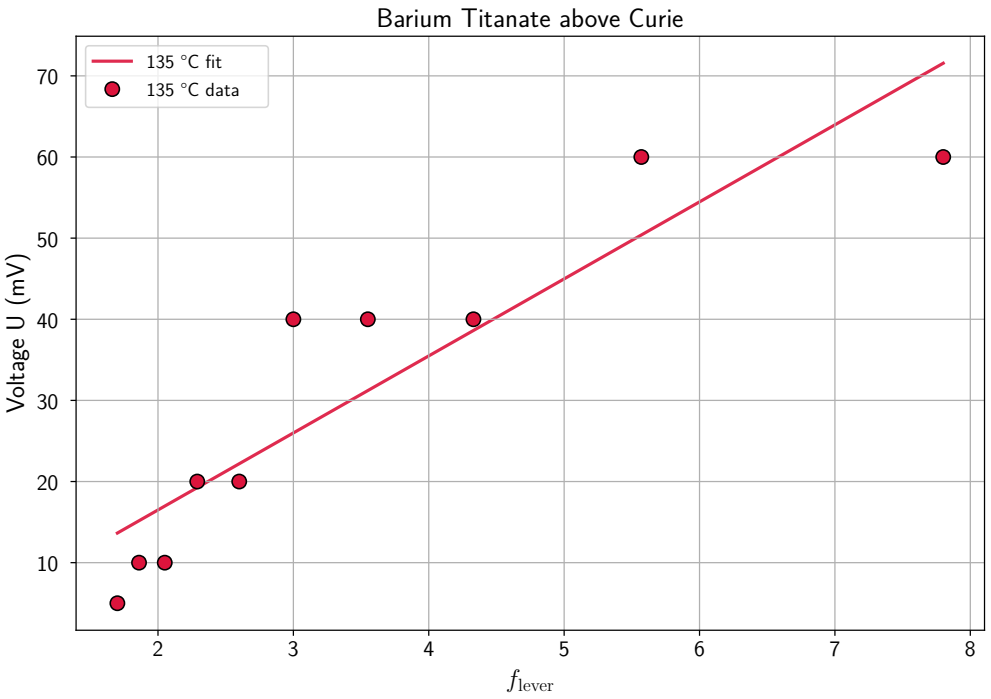


Figure 4: Measured voltage U as a function of the lever factor f_{lever} for barium titanate above the Curie temperature.

The measured coefficient above the Curie temperature was found to be

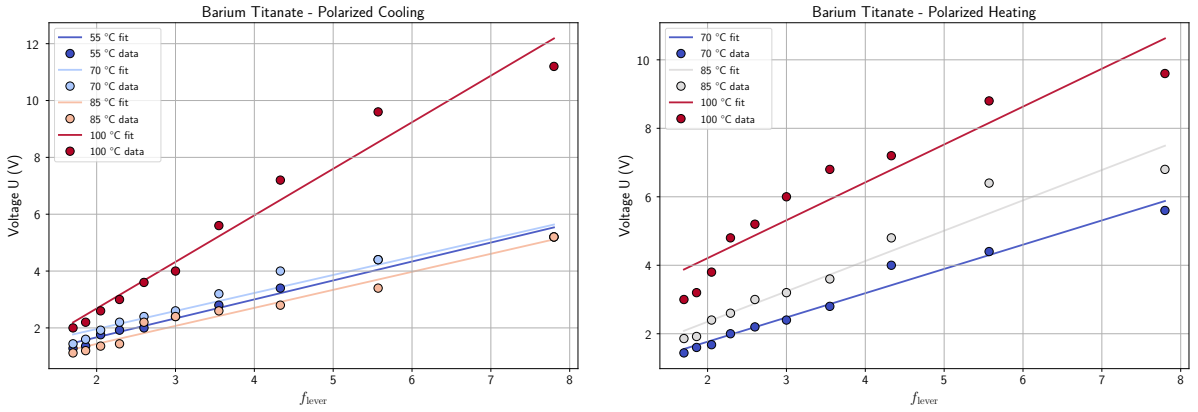
$$d_{11}(135°\text{C}) = 8.69 \times 10^{-13}\ \text{C}\,\text{N}^{-1}. \tag{32}$$

In the ideal cubic phase, the piezoelectric coefficient should vanish, i.e. $d_{11} \approx 0$. The measured value is therefore close to the expected result and is significantly smaller than the values obtained below the Curie temperature. This confirms the expected suppression of piezoelectricity in the high-temperature phase. The remaining small non-zero value can be attributed to the finite fit quality, residual voltage offsets, possible temperature gradients inside the sample, and the limited accuracy of reading the weak oscilloscope

signal. Since the temperature was measured at the oven thermometers, the crystal may not have been perfectly homogeneous in temperature immediately after heating, so small cooler regions could still contribute weakly to the measured signal.

4.3 Barium Titanate: Polarized Sample

For the polarized barium titanate sample, the measured voltages are much larger than for quartz and also much larger than for the unpolarized barium titanate sample. This reflects the fact that polarization aligns the ferroelectric domains and therefore increases the net piezoelectric response of the ceramic.



(a) Polarized sample during cooling.

(b) Polarized sample during heating.

Figure 5: Voltage U as a function of the lever factor f_{lever} for polarized barium titanate during cooling and heating.

For the polarized cooling series, the coefficients were found to be

$$d_{11}(55°\text{C}) = 6.10 \times 10^{-11}\ \text{C}\,\text{N}^{-1}, \tag{33}$$

$$d_{11}(70°\text{C}) = 5.80 \times 10^{-11}\ \text{C}\,\text{N}^{-1}, \tag{34}$$

$$d_{11}(85°\text{C}) = 5.80 \times 10^{-11}\ \text{C}\,\text{N}^{-1}, \tag{35}$$

$$d_{11}(100°\text{C}) = 1.50 \times 10^{-10}\ \text{C}\,\text{N}^{-1}. \tag{36}$$

For the polarized heating series, the values are

$$d_{11}(70°\text{C}) = 6.48 \times 10^{-11}\ \text{C}\,\text{N}^{-1}, \tag{37}$$

$$d_{11}(85°\text{C}) = 8.11 \times 10^{-11}\ \text{C}\,\text{N}^{-1}, \tag{38}$$

$$d_{11}(100°\text{C}) = 1.01 \times 10^{-10}\ \text{C}\,\text{N}^{-1}. \tag{39}$$

These values are more than one order of magnitude larger than those obtained for quartz, which is consistent with the strong piezoelectric response expected for polarized ferroelectric ceramics. The heating and cooling data are of the same order of magnitude, but not

identical. This is plausible because barium titanate can exhibit hysteresis near the ferroelectric phase transition: the domain configuration depends not only on the temperature itself but also on the thermal history of the sample.

The largest response is observed near 100°C, i.e. relatively close to the Curie temperature but still below it. This suggests that the piezoelectric response is enhanced in this temperature region, while it drops sharply above the transition.

For barium titanate, a direct comparison with a single literature value for d_{11} is less straightforward than for quartz. In the literature, $BaTiO_3$ ceramics are usually characterized by the standard tensor components d_{33}, d_{31}, and d_{15}. Typical values reported for $BaTiO_3$ ceramics are, for example, $d_{33} = 191\,\mathrm{pC\,N^{-1}}$, $d_{31} = -79\,\mathrm{pC\,N^{-1}}$, and $d_{15} = 270\,\mathrm{pC\,N^{-1}}$ [2]. The coefficient determined in the present experiment is an effective coefficient in the specific measurement geometry and is denoted d_{11} following the experiment manual. Therefore, the comparison with literature values should be understood as an order-of-magnitude comparison rather than a direct comparison of identical tensor components.

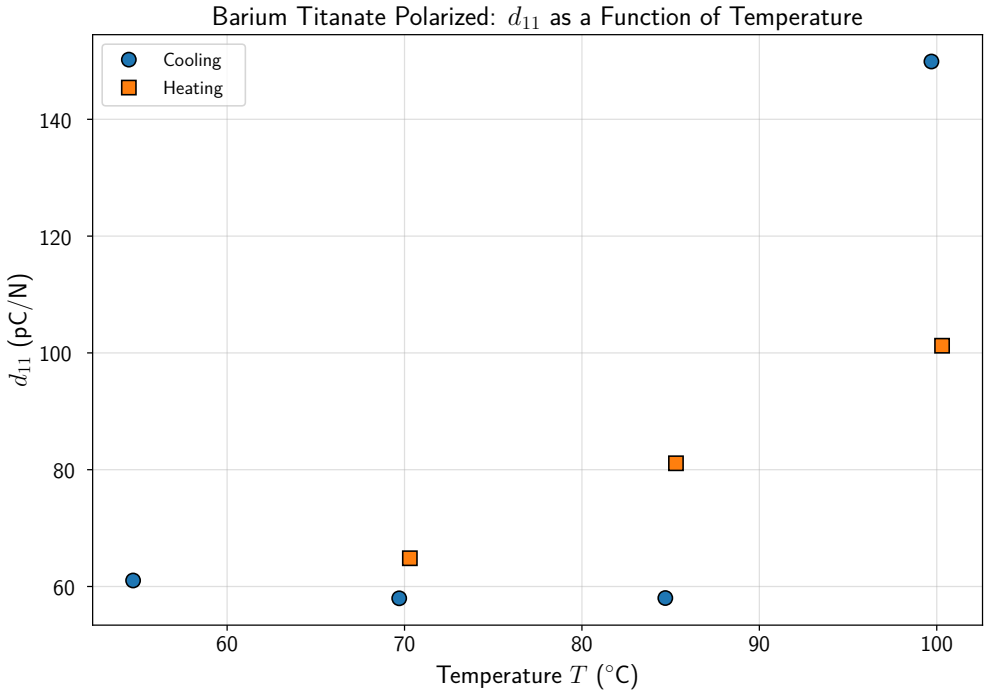


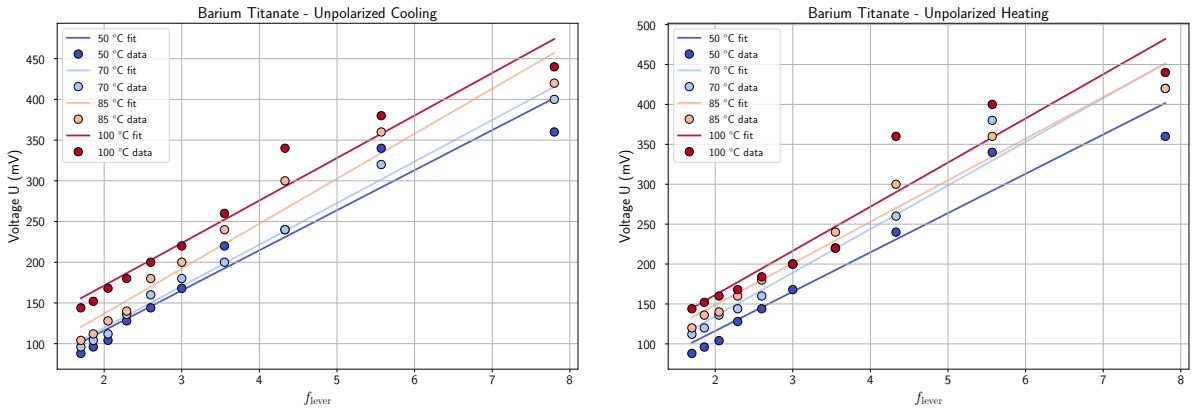
Figure 6: Piezoelectric coefficient d_{11} of polarized barium titanate as a function of temperature during heating and cooling. The heating and cooling points are slightly shifted horizontally for visibility.

Figure 6 summarizes the temperature dependence of the extracted piezoelectric coefficient for the polarized barium titanate sample. The heating series shows an approximately increasing response when approaching the Curie temperature from below. The cooling series is of the same order of magnitude, but differs from the heating series, which is consistent with thermal-history effects and hysteresis near the ferroelectric phase transition. Since

only a few temperature points were measured, no trend function was fitted; the plot is used only to visualize the measured temperature dependence.

4.4 Barium Titanate: Unpolarized Sample

The unpolarized barium titanate sample also exhibits a measurable piezoelectric response below the Curie temperature, but the signal is much weaker than in the polarized case.



(a) Unpolarized sample during cooling.

(b) Unpolarized sample during heating.

Figure 7: Voltage U as a function of the lever factor f_{lever} for unpolarized barium titanate during cooling and heating.

For the unpolarized cooling series, the fitted coefficients are

$$d_{11}(50°\text{C}) = 4.50 \times 10^{-12}\ \text{C}\,\text{N}^{-1}, \tag{40}$$
$$d_{11}(70°\text{C}) = 4.67 \times 10^{-12}\ \text{C}\,\text{N}^{-1}, \tag{41}$$
$$d_{11}(85°\text{C}) = 5.06 \times 10^{-12}\ \text{C}\,\text{N}^{-1}, \tag{42}$$
$$d_{11}(100°\text{C}) = 4.78 \times 10^{-12}\ \text{C}\,\text{N}^{-1}. \tag{43}$$

For the unpolarized heating series, the values are

$$d_{11}(50°\text{C}) = 4.50 \times 10^{-12}\ \text{C}\,\text{N}^{-1}, \tag{44}$$
$$d_{11}(70°\text{C}) = 5.00 \times 10^{-12}\ \text{C}\,\text{N}^{-1}, \tag{45}$$
$$d_{11}(85°\text{C}) = 4.76 \times 10^{-12}\ \text{C}\,\text{N}^{-1}, \tag{46}$$
$$d_{11}(100°\text{C}) = 5.06 \times 10^{-12}\ \text{C}\,\text{N}^{-1}. \tag{47}$$

These values are clearly larger than the above-Curie result, showing that the sample is still piezoelectric below the phase transition. However, they are much smaller than for the polarized sample. This is expected, since in an ideal unpolarized ceramic the ferroelectric domains are randomly oriented and their contributions cancel to a large extent,

resulting in no macroscopic piezoelectric activity [2]. Therefore, the ideal expectation for an unpolarized ceramic is $d_{11} \approx 0$.

The small but non-zero values measured here, $d_{11} \approx 4.5$–$5.1\,\mathrm{pC\,N^{-1}}$, may indicate that the sample was not perfectly unpolarized. Possible reasons include residual polarization from incomplete depolarization, thermal history effects, and small mechanical stresses introduced during mounting of the sample. Compared with the polarized sample, the unpolarized sample also shows much less variation between heating and cooling. The values at 50 °C are identical for the heating and cooling series, since the same data points were used for this temperature.

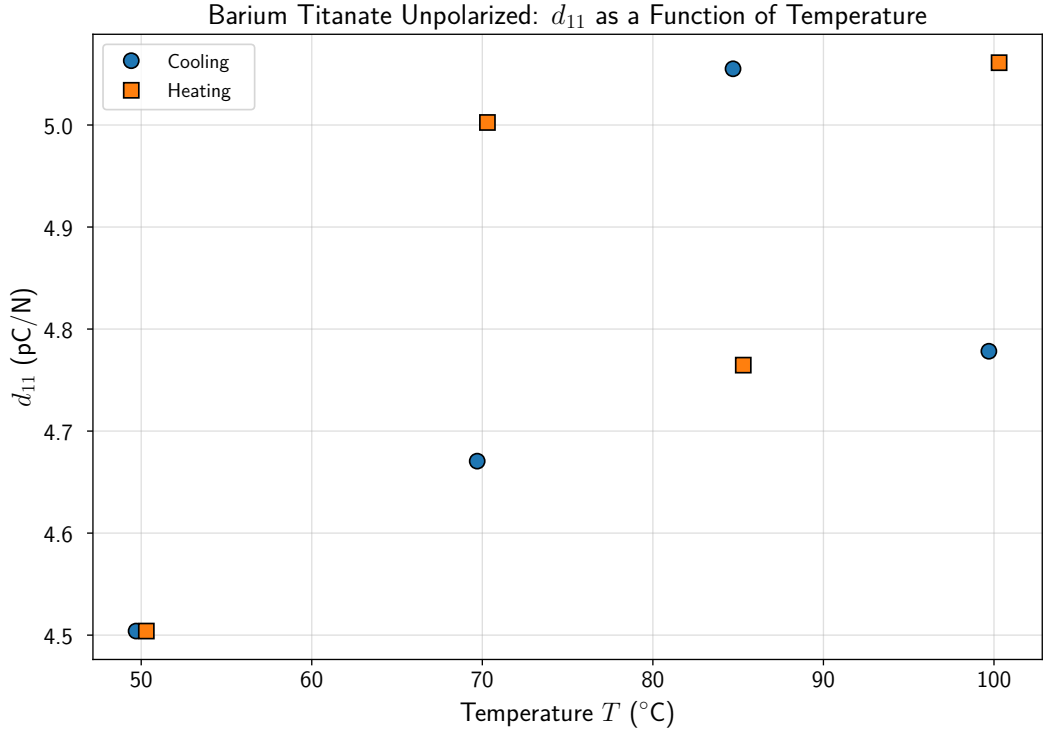


Figure 8: Piezoelectric coefficient d_{11} of unpolarized barium titanate as a function of temperature during heating and cooling. The heating and cooling points are slightly shifted horizontally for visibility.

Figure 8 summarizes the temperature dependence of the extracted piezoelectric coefficient for the unpolarized barium titanate sample. The values remain in a narrow range of approximately 4.5–$5.1\,\mathrm{pC\,N^{-1}}$. Compared with the polarized sample, the response is much smaller and shows less pronounced variation between heating and cooling. This supports the interpretation that the domains in the unpolarized ceramic are largely randomly oriented, so their macroscopic piezoelectric contributions mostly cancel. Since only a small number of temperature points were measured and no clear systematic trend is visible, no trend function was fitted.

4.5 Error Analysis

To estimate the influence of measurement scatter and the linear regression on the final value of d_{11}, the quartz measurement at 24°C was used as a representative example. The data were fitted with

$$U = M f_{\text{lever}} + b. \tag{48}$$

For this measurement, the fit yielded

$$M = (30.7 \pm 2.6)\,\text{mV}, \tag{49}$$

where the uncertainty is the standard error of the fitted slope. Since

$$d_{11} = \frac{\pi}{2RK_0\omega}\, M, \tag{50}$$

the uncertainty of d_{11} due to the regression is obtained by linear error propagation:

$$\Delta d_{11} = \frac{\pi}{2RK_0\omega}\, \Delta M. \tag{51}$$

Here, the slope was converted from mV to V before calculating d_{11}. This gives

$$d_{11}(24°\text{C}) = (2.81 \pm 0.24) \times 10^{-12}\ \text{C}\,\text{N}^{-1}. \tag{52}$$

This uncertainty only includes the statistical uncertainty from the linear regression. Additional systematic uncertainties arise from the voltage reading on the oscilloscope, the determination of the force amplitude K_0, the oscillation frequency, the exact release elongation, and possible temperature variations during the measurement. In particular, damping of the spring oscillation after release can lead to deviations from a perfectly constant force amplitude. Therefore, the quoted uncertainty should be interpreted as the regression uncertainty rather than the total experimental uncertainty.

4.6 Comparison and Physical Interpretation

The results show a clear hierarchy of the measured effective piezoelectric response:

$$d_{11}(\text{polarized BaTiO}_3) \gg d_{11}(\text{unpolarized BaTiO}_3) \gtrsim d_{11}(\text{quartz}) > d_{11}(\text{above Curie}). \tag{53}$$

The polarized barium titanate sample shows the strongest response, while the above-Curie measurement gives the smallest value, close to the expected disappearance of piezoelectricity in the cubic phase.

This behavior is consistent with the microscopic picture of the materials. Quartz is a non-centrosymmetric crystal with a comparatively stable structure, so its piezoelectric

coefficient changes only weakly with temperature. Barium titanate, by contrast, is ferroelectric below the Curie temperature. In the polarized sample, a preferred domain orientation is produced, which strongly enhances the macroscopic piezoelectric response. In the unpolarized sample, the domains remain more randomly distributed, so their contributions cancel to a large extent, and only a small effective response remains. Once the Curie temperature is exceeded, the ferroelectric order disappears, and the piezoelectric effect is strongly suppressed.

The fitted relations are generally close to linear, which supports the assumed proportionality between U and f_{lever}. Nevertheless, several fits show non-zero intercepts and some scatter. Possible reasons include residual voltage offsets, finite oscilloscope reading accuracy, uncertainties in the force amplitude, damping of the spring oscillation after release, and temperature variations during the measurements. The above-Curie measurement shows the weakest signal and the lowest fit quality, which is consistent with the expectation that the piezoelectric response is nearly absent in that regime.

5 Conclusion

In this experiment, the direct piezoelectric effect was investigated for quartz and barium titanate by measuring the voltage generated under an oscillating mechanical force. For each temperature, the measured voltage was plotted as a function of the lever factor f_{lever}, and the piezoelectric coefficient d_{11} was determined from the slope of a linear fit. The observed approximately linear dependence confirms the expected proportionality between mechanical excitation and electrical response.

For quartz, the extracted values of d_{11} remained within a narrow range over the investigated temperatures. The average value was found to be

$$\bar{d_{11}} = (2.87 \pm 0.10) \times 10^{-12}\ \mathrm{C\,N^{-1}}, \tag{54}$$

where the uncertainty denotes the standard error of the mean. This confirms that quartz has only a weak temperature dependence of its piezoelectric coefficient and is therefore suitable as a stable reference material. The value is also in reasonable agreement with the literature value for quartz.

Barium titanate showed a much stronger dependence on temperature and polarization state. The polarized sample produced significantly larger piezoelectric coefficients than the unpolarized sample, demonstrating the importance of ferroelectric domain alignment for the macroscopic piezoelectric response. In the unpolarized sample, the randomly oriented domains largely cancel each other, resulting in a much smaller effective coefficient. Above the Curie temperature, the measured response was strongly reduced and close to zero, consistent with the transition to the paraelectric cubic phase, where the piezoelectric effect is suppressed.

The main sources of uncertainty were the scatter of the data points in the linear fits, the finite accuracy of the voltage readings, the determination of the force amplitude K_0 and oscillation frequency, temperature variations, and damping of the oscillating spring system. Despite these limitations, the measurements reproduce the central theoretical expectations: quartz exhibits a stable piezoelectric response, whereas barium titanate shows a strong dependence on temperature and ferroelectric domain alignment.

Appendix

Acknowledgments

I would like to thank my teaching assistant, Dr. Konrad Puzniak, for his supervision and support during the experiment. His guidance and explanations were very helpful throughout the laboratory work and the interpretation of the results.

AI Usage Declaration

This report was prepared with assistance from ChatGPT (OpenAI) for language polishing, formatting suggestions, and for helping to develop, debug, and correct the syntax of the Python code. The AI was not used to generate raw data, to select results, or to determine the final conclusions. All scientific content (methods, calculations, results, and interpretation) was critically reviewed and validated by the author. The author takes full responsibility for the accuracy of the analysis and for any remaining errors.

Safety Measures

The setup was already properly grounded before the measurements, so no additional grounding was required. The crystals were not touched with bare hands in order to avoid contamination of the surfaces; gloves were worn and the samples were cleaned with alcohol before use. The main possible safety risk was the heating oven, since temperatures above 100 °C were reached during the experiment. However, the risk was low as long as direct contact with the hot oven or heated metal parts was avoided.

References

[1] Boston Piezo-Optics Inc. *Crystal Quartz Properties*. https://www.bostonpiezooptics.com/crystal-quartz. Material data page, piezoelectric and dielectric properties; accessed 10 May 2026.

[2] Vincenzo Buscaglia, Maria Teresa Buscaglia, and Giovanna Canu. "BaTiO3-Based Ceramics: Fundamentals, Properties and Applications". In: *Encyclopedia of Materials: Technical Ceramics and Glasses*. Elsevier, 2021, pp. 311–344. ISBN: 978-0-12-822233-1. DOI: 10.1016/B978-0-12-803581-8.12132-0. URL: https://iris.cnr.it/retrieve/b6f46208-86ef-4642-bd45-35d5ef17fcd0/prod_470988-doc_191164.pdf.

[3] "Fundamentals of Piezoelectricity". In: *Lead-Free Piezoelectric Materials*. John Wiley & Sons, Ltd, 2021. Chap. 1, pp. 1–18. ISBN: 9783527817047. DOI: 10.1002/9783527817047.ch1. URL: https://onlinelibrary.wiley.com/doi/abs/10.1002/9783527817047.ch1.

[4] M. G. Harwood, P. Popper, and D. F. Rushman. "Curie Point of Barium Titanate". In: *Nature* 160 (July 1947), pp. 58–59. DOI: 10.1038/160058a0. URL: https://www.nature.com/articles/160058a0.

[5] Manuel Schneider. *Piezoelectric Effect*. Physics Laboratory 3 & 4 Manual. ETH Zürich. Zürich, Switzerland, May 2011.

Raw and calculated data

This section contains the raw measurement data and the calculated intermediate quantities used in the analysis. For each measurement series, the table lists the temperature T, the lever setting x, the measured voltage U, the lever factor f_{lever}, and the force K_x acting on the crystal.

Quartz Crystal

Table 2: Raw and calculated data for quartz at different temperatures.

Temperature T (°C)	Lever setting x (mm)	Voltage U (mV)	Lever factor f_{lever}	Force K_x (N)
24	30	240	7.80	308
24	70	220	5.57	220
24	110	180	4.33	171
24	150	136	3.55	140
24	190	120	3.00	119
24	230	104	2.60	103
24	270	92	2.29	90.6
24	310	84	2.05	81.1
24	350	76	1.86	73.4
24	390	68	1.70	67.0
62	30	260	7.80	308
62	70	200	5.57	220
62	110	168	4.33	171
62	150	136	3.55	140
62	190	120	3.00	119
62	230	96	2.60	103
62	270	92	2.29	90.6
62	310	84	2.05	81.1
62	350	72	1.86	73.4
62	390	64	1.70	67.0

Continued on next page

Table 2: Raw and calculated data for quartz at different temperatures. (Continued)

Temperature T (°C)	Lever setting x (mm)	Voltage U (mV)	Lever factor f_{lever}	Force K_x (N)
99	30	240	7.80	308
99	70	180	5.57	220
99	110	160	4.33	171
99	150	128	3.55	140
99	190	112	3.00	119
99	230	96	2.60	103
99	270	88	2.29	90.6
99	310	80	2.05	81.1
99	350	72	1.86	73.4
99	390	64	1.70	67.0
124	30	250	7.80	308
124	70	210	5.57	220
124	110	170	4.33	171
124	150	130	3.55	140
124	190	110	3.00	119
124	230	90	2.60	103
124	270	80	2.29	90.6
124	310	72	2.05	81.1
124	350	64	1.86	73.4
124	390	56	1.70	67.0

Barium Titanate Crystal

Curie Temperature Measurements

Table 3: Raw and calculated data for polarized barium titanate above the Curie temperature.

Lever setting x (mm)	Voltage U (mV)	Lever factor f_{lever}	Force K_x (N)
30	60	7.80	308
70	60	5.57	220
110	40	4.33	171
150	40	3.55	140
190	40	3.00	119
230	20	2.60	103
270	20	2.29	90.6
310	10	2.05	81.1
350	10	1.86	73.4
390	05	1.70	67.0

Barium Titanate: Polarized Sample

Table 4: Raw and calculated data for polarized barium titanate during cooling.

Temperature T (°C)	Lever setting x (mm)	Voltage U (V)	Lever factor f_{lever}	Force K_x (N)
100	30	11.2	7.80	308
100	70	9.6	5.57	220
100	110	7.2	4.33	171
100	150	5.6	3.55	140
100	190	4.0	3.00	119
100	230	3.6	2.60	103
100	270	3.0	2.29	90.6
100	310	2.6	2.05	81.1

Continued on next page

Table 4: Raw and calculated data for polarized barium titanate during cooling. (Continued)

Temperature T (°C)	Lever setting x (mm)	Voltage U (V)	Lever factor f_{lever}	Force K_x (N)
100	350	2.2	1.86	73.4
100	390	2.0	1.70	67.0
85	30	5.2	7.80	308
85	70	3.4	5.57	220
85	110	2.8	4.33	171
85	150	2.6	3.55	140
85	190	2.4	3.00	119
85	230	2.2	2.60	103
85	270	1.44	2.29	90.6
85	310	1.36	2.05	81.1
85	350	1.20	1.86	73.4
85	390	1.12	1.70	67.0
70	30	5.2	7.80	308
70	70	4.4	5.57	220
70	110	4.0	4.33	171
70	150	3.2	3.55	140
70	190	2.6	3.00	119
70	230	2.4	2.60	103
70	270	2.2	2.29	90.6
70	310	1.92	2.05	81.1
70	350	1.60	1.86	73.4
70	390	1.44	1.70	67.0
55	30	5.2	7.80	308
55	70	4.4	5.57	220
55	110	3.4	4.33	171
55	150	2.8	3.55	140
55	190	2.4	3.00	119

Continued on next page

Table 4: Raw and calculated data for polarized barium titanate during cooling. (Continued)

Temperature T (°C)	Lever setting x (mm)	Voltage U (V)	Lever factor f_{lever}	Force K_x (N)
55	230	2.0	2.60	103
55	270	1.92	2.29	90.6
55	310	1.76	2.05	81.1
55	350	1.36	1.86	73.4
55	390	1.28	1.70	67.0

Table 5: Raw and calculated data for polarized barium titanate during heating.

Temperature T (°C)	Lever setting x (mm)	Voltage U (V)	Lever factor f_{lever}	Force K_x (N)
70	30	5.6	7.80	308
70	70	4.4	5.57	220
70	110	4.0	4.33	171
70	150	2.8	3.55	140
70	190	2.4	3.00	119
70	230	2.2	2.60	103
70	270	2.0	2.29	90.6
70	310	1.68	2.05	81.1
70	350	1.60	1.86	73.4
70	390	1.44	1.70	67.0
85	30	6.8	7.80	308
85	70	6.4	5.57	220
85	110	4.8	4.33	171
85	150	3.6	3.55	140
85	190	3.2	3.00	119
85	230	3.0	2.60	103
85	270	2.6	2.29	90.6

Continued on next page

Table 5: Raw and calculated data for polarized barium titanate during heating. (Continued)

Temperature T (°C)	Lever setting x (mm)	Voltage U (V)	Lever factor f_{lever}	Force K_x (N)
85	310	2.4	2.05	81.1
85	350	1.92	1.86	73.4
85	390	1.86	1.70	67.0
100	30	9.6	7.80	308
100	70	8.8	5.57	220
100	110	7.2	4.33	171
100	150	6.8	3.55	140
100	190	6.0	3.00	119
100	230	5.2	2.60	103
100	270	4.8	2.29	90.6
100	310	3.8	2.05	81.1
100	350	3.2	1.86	73.4
100	390	3.0	1.70	67.0

Barium Titanate: Unpolarized Sample

Table 6: Raw and calculated data for unpolarized barium titanate during cooling.

Temperature T (°C)	Lever setting x (mm)	Voltage U (mV)	Lever factor f_{lever}	Force K_x (N)
100	30	440	7.80	308
100	70	380	5.57	220
100	110	340	4.33	171
100	150	260	3.55	140
100	190	220	3.00	119
100	230	200	2.60	103
100	270	180	2.29	90.6
100	310	168	2.05	81.1

Continued on next page

Table 6: Raw and calculated data for unpolarized barium titanate during cooling. (Continued)

Temperature T (°C)	Lever setting x (mm)	Voltage U (mV)	Lever factor f_{lever}	Force K_x (N)
100	350	152	1.86	73.4
100	390	144	1.70	67.0
85	30	420	7.80	308
85	70	360	5.57	220
85	110	300	4.33	171
85	150	240	3.55	140
85	190	200	3.00	119
85	230	180	2.60	103
85	270	140	2.29	90.6
85	310	128	2.05	81.1
85	350	112	1.86	73.4
85	390	104	1.70	67.0
70	30	400	7.80	308
70	70	320	5.57	220
70	110	240	4.33	171
70	150	200	3.55	140
70	190	180	3.00	119
70	230	160	2.60	103
70	270	136	2.29	90.6
70	310	112	2.05	81.1
70	350	104	1.86	73.4
70	390	96	1.70	67.0
50	30	360	7.80	308
50	70	340	5.57	220
50	110	240	4.33	171
50	150	220	3.55	140
50	190	168	3.00	119

Continued on next page

Table 6: Raw and calculated data for unpolarized barium titanate during cooling. (Continued)

Temperature T (°C)	Lever setting x (mm)	Voltage U (mV)	Lever factor f_{lever}	Force K_x (N)
50	230	144	2.60	103
50	270	128	2.29	90.6
50	310	104	2.05	81.1
50	350	96	1.86	73.4
50	390	88	1.70	67.0

Table 7: Raw and calculated data for unpolarized barium titanate during heating.

Temperature T (°C)	Lever setting x (mm)	Voltage U (mV)	Lever factor f_{lever}	Force K_x (N)
50	30	360	7.80	308
50	70	340	5.57	220
50	110	240	4.33	171
50	150	220	3.55	140
50	190	168	3.00	119
50	230	144	2.60	103
50	270	128	2.29	90.6
50	310	104	2.05	81.1
50	350	96	1.86	73.4
50	390	88	1.70	67.0
70	30	420	7.80	308
70	70	380	5.57	220
70	110	260	4.33	171
70	150	220	3.55	140
70	190	200	3.00	119
70	230	160	2.60	103
70	270	144	2.29	90.6

Continued on next page

Table 7: Raw and calculated data for unpolarized barium titanate during heating. (Continued)

Temperature T (°C)	Lever setting x (mm)	Voltage U (mV)	Lever factor f_{lever}	Force K_x (N)
70	310	136	2.05	81.1
70	350	120	1.86	73.4
70	390	112	1.70	67.0
85	30	420	7.80	308
85	70	360	5.57	220
85	110	300	4.33	171
85	150	240	3.55	140
85	190	200	3.00	119
85	230	180	2.60	103
85	270	160	2.29	90.6
85	310	140	2.05	81.1
85	350	136	1.86	73.4
85	390	120	1.70	67.0
100	30	440	7.80	308
100	70	400	5.57	220
100	110	360	4.33	171
100	150	220	3.55	140
100	190	200	3.00	119
100	230	184	2.60	103
100	270	168	2.29	90.6
100	310	160	2.05	81.1
100	350	152	1.86	73.4
100	390	144	1.70	67.0

Python Code

1. Spring Constant.py

```python
import numpy as np

# ----------------------------
# Input data
# ----------------------------
l0_mm = 65.0  # unloaded spring length in mm
g = 9.81      # gravitational acceleration in m/s^2

masses_g = np.array([472, 1000, 2003, 4023], dtype=float)   # masses in g
lengths_mm = np.array([67, 69, 89, 143], dtype=float)       # spring lengths in mm

# ----------------------------
# Calculations
# ----------------------------
elongation_mm = lengths_mm - l0_mm
elongation_m = elongation_mm / 1000.0
masses_kg = masses_g / 1000.0
forces_N = masses_kg * g
k_values = forces_N / elongation_m

# ----------------------------
# Statistics
# ----------------------------
k_mean = np.mean(k_values)
k_std = np.std(k_values, ddof=1)          # sample standard deviation
k_sem = k_std / np.sqrt(len(k_values))    # standard error of the mean

# ----------------------------
# Output
# ----------------------------
print("Spring calibration results")
print("-" * 72)
print(f"{'m (g)':>8} {'l (mm)':>8} {'dL (mm)':>10} {'F (N)':>10} {'k (N/m)':>12}")
print("-" * 72)

```

```
for m, l, dl, F, k in zip(masses_g, lengths_mm, elongation_mm, forces_N,
    k_values):
     print(f"{m:8.1f} {l:8.1f} {dl:10.1f} {F:10.3f} {k:12.2f}")

print("-" * 72)
print(f"Mean spring constant: k = {k_mean:.2f} pm {k_std:.2f} N/m   (standard
    deviation)")
print(f"Mean spring constant: k = {k_mean:.2f} pm {k_sem:.2f} N/m   (standard
    error)")
```

2. Raw Data Calculation.py

```python
# ============================================
# Piezoelectric Effect Lab
# Calculation of lever factor f_lever and force K_x
# from the raw measurement data
# ============================================

# ------------------------------------------------
# 1. Constants used in the whole script
# ------------------------------------------------

X0_MM = 30.0                    # x0 in mm
K0_N = 39.5                     # force amplitude in N, from spring calibration
# K_x = f_lever * K0

# ------------------------------------------------
# 2. Helper functions
# ------------------------------------------------

def compute_f_lever(x_mm: float) -> float:
    """
    Compute the lever factor from the lever setting x.

    Formula:
        f_lever = 780 / (100 + (x - x0))

    with x0 = 30 mm.
    """
    return 780.0 / (100.0 + (x_mm - X0_MM))


def compute_force_kx(x_mm: float) -> float:
    """
    Compute the force K_x in Newton from x.

    Formula:
        K_x = f_lever * K0
    """
    f_lever = compute_f_lever(x_mm)
    return f_lever * K0_N

```

```
41
42  def print_series(title: str, data: dict, voltage_unit: str):
43      """
44      Print one complete measurement series in a structured way.
45
46      Parameters
47      ----------
48      title : str
49          Title of the series, e.g. 'Quartz Crystal'
50      data : dict
51          Dictionary of the form
52          {
53              temperature: [(x1, U1), (x2, U2), ...],
54              ...
55          }
56      voltage_unit : str
57          Either 'mV' or 'V'
58      """
59      print("\n" + "=" * 90)
60      print(title)
61      print("=" * 90)
62
63      for temperature, rows in data.items():
64          print(f"\nTemperature = {temperature} deg C")
65          print("-" * 90)
66          print(f"{'x (mm)':>8} {'U (' + voltage_unit + ')':>12} {'f_lever':>12}
    {'K_x (N)':>12}")
67          print("-" * 90)
68
69          for x_mm, voltage in rows:
70              f_lever = compute_f_lever(x_mm)
71              kx = compute_force_kx(x_mm)
72              print(f"{x_mm:8.0f} {voltage:12.3f} {f_lever:12.3f} {kx:12.3f}")
73
74
75  def print_curie_series(title: str, rows: list, voltage_unit: str):
76      """
77      Print the Above Curie measurement series (without temperature column).
78      """
79      print("\n" + "=" * 90)
80      print(title)
```

```
    print("=" * 90)
    print(f"{'x (mm)':>8} {'U (' + voltage_unit + ')':>12} {'f_lever':>12} {'K_x (N)':>12}")
    print("-" * 90)

    for x_mm, voltage in rows:
        f_lever = compute_f_lever(x_mm)
        kx = compute_force_kx(x_mm)
        print(f"{x_mm:8.0f} {voltage:12.3f} {f_lever:12.3f} {kx:12.3f}")


# ------------------------------------------------
# 3. Raw data: Quartz Crystal
#    Voltage unit: mV
# ------------------------------------------------

quartz_data = {
    24: [
        (30, 240), (70, 220), (110, 180), (150, 136), (190, 120),
        (230, 104), (270, 92), (310, 84), (350, 76), (390, 68)
    ],
    62: [
        (30, 260), (70, 200), (110, 168), (150, 136), (190, 120),
        (230, 96), (270, 92), (310, 84), (350, 72), (390, 64)
    ],
    99: [
        (30, 240), (70, 180), (110, 160), (150, 128), (190, 112),
        (230, 96), (270, 88), (310, 80), (350, 72), (390, 64)
    ],
    124: [
        (30, 250), (70, 210), (110, 170), (150, 130), (190, 110),
        (230, 90), (270, 80), (310, 72), (350, 64), (390, 56)
    ]
}

# ------------------------------------------------
# 4. Raw data: Barium Titanate above Curie
#    Voltage unit: mV
# ------------------------------------------------

curie_data = [
```

```
    (30, 60), (70, 60), (110, 40), (150, 40), (190, 40),
    (230, 20), (270, 20), (310, 10), (350, 10), (390, 5)
]

# ------------------------------------------------
# 5. Raw data: Barium Titanate polarized cooling
#    Voltage unit: V
# ------------------------------------------------

bt_pol_cooling = {
    100: [
        (30, 11.2), (70, 9.6), (110, 7.2), (150, 5.6), (190, 4.0),
        (230, 3.6), (270, 3.0), (310, 2.6), (350, 2.2), (390, 2.0)
    ],
    85: [
        (30, 5.2), (70, 3.4), (110, 2.8), (150, 2.6), (190, 2.4),
        (230, 2.2), (270, 1.44), (310, 1.36), (350, 1.20), (390, 1.12)
    ],
    70: [
        (30, 5.2), (70, 4.4), (110, 4.0), (150, 3.2), (190, 2.6),
        (230, 2.4), (270, 2.2), (310, 1.92), (350, 1.60), (390, 1.44)
    ],
    55: [
        (30, 5.2), (70, 4.4), (110, 3.4), (150, 2.8), (190, 2.4),
        (230, 2.0), (270, 1.92), (310, 1.76), (350, 1.36), (390, 1.28)
    ]
}

# ------------------------------------------------
# 6. Raw data: Barium Titanate polarized heating
#    Voltage unit: V
# ------------------------------------------------

bt_pol_heating = {
    70: [
        (30, 5.6), (70, 4.4), (110, 4.0), (150, 2.8), (190, 2.4),
        (230, 2.2), (270, 2.0), (310, 1.68), (350, 1.60), (390, 1.44)
    ],
    85: [
        (30, 6.8), (70, 6.4), (110, 4.8), (150, 3.6), (190, 3.2),
        (230, 3.0), (270, 2.6), (310, 2.4), (350, 1.92), (390, 1.86)
```

```
    ],
    100: [
        (30, 9.6), (70, 8.8), (110, 7.2), (150, 6.8), (190, 6.0),
        (230, 5.2), (270, 4.8), (310, 3.8), (350, 3.2), (390, 3.0)
    ]
}

# ------------------------------------------------
# 7. Raw data: Barium Titanate unpolarized cooling
#    Voltage unit: mV
# ------------------------------------------------

bt_unpol_cooling = {
    100: [
        (30, 440), (70, 380), (110, 340), (150, 260), (190, 220),
        (230, 200), (270, 180), (310, 168), (350, 152), (390, 144)
    ],
    85: [
        (30, 420), (70, 360), (110, 300), (150, 240), (190, 200),
        (230, 180), (270, 140), (310, 128), (350, 112), (390, 104)
    ],
    70: [
        (30, 400), (70, 320), (110, 240), (150, 200), (190, 180),
        (230, 160), (270, 136), (310, 112), (350, 104), (390, 96)
    ],
    50: [
        (30, 360), (70, 340), (110, 240), (150, 220), (190, 168),
        (230, 144), (270, 128), (310, 104), (350, 96), (390, 88)
    ]
}

# ------------------------------------------------
# 8. Raw data: Barium Titanate unpolarized heating
# ------------------------------------------------

bt_unpol_heating = {
    50: [
        (30, 360), (70, 340), (110, 240), (150, 220), (190, 168),
        (230, 144), (270, 128), (310, 104), (350, 96), (390, 88)
    ],
    70: [
```

```
        (30, 420), (70, 380), (110, 260), (150, 220), (190, 200),
        (230, 160), (270, 144), (310, 136), (350, 120), (390, 112)
    ],
    85: [
        (30, 420), (70, 360), (110, 300), (150, 240), (190, 200),
        (230, 180), (270, 160), (310, 140), (350, 136), (390, 120)
    ],
    100: [
        (30, 440), (70, 400), (110, 360), (150, 220), (190, 200),
        (230, 184), (270, 168), (310, 160), (350, 152), (390, 144)
    ]
}

# ------------------------------------------------
# 9. Run all outputs
# ------------------------------------------------

print("\nCalculated quantities for all raw data")
print("Used constants:")
print(f"  x0   = {X0_MM:.1f} mm")
print(f"  K0   = {K0_N:.1f} N")
print("  Formula for lever factor: f_lever = 780 / (100 + (x - x0))")
print("  Formula for force:        K_x = f_lever * K0")

print_series("Quartz Crystal", quartz_data, "mV")
print_curie_series("Barium Titanate above Curie temperature", curie_data, "mV")
print_series("Barium Titanate - Polarized Cooling", bt_pol_cooling, "V")
print_series("Barium Titanate - Polarized Heating", bt_pol_heating, "V")
print_series("Barium Titanate - Unpolarized Cooling", bt_unpol_cooling, "mV")
print_series("Barium Titanate - Unpolarized Heating", bt_unpol_heating, "mV")
```

3. Plots.py

```python
import numpy as np
import matplotlib.pyplot as plt
from matplotlib import cm

# ------------------------------------------------------------
# Global plot style
# ------------------------------------------------------------
plt.rcParams.update({
    "font.size": 15,
    "axes.titlesize": 17,
    "axes.labelsize": 16,
    "xtick.labelsize": 14,
    "ytick.labelsize": 14,
    "legend.fontsize": 12,
    "lines.linewidth": 2.2,
    "lines.markersize": 8,
})


# ============================================================
# PIEZOELECTRIC EFFECT LAB
# Linear fits of U versus f_lever and determination of d_11
# ============================================================

# ------------------------------------------------------------
# 1. Physical constants from the report
# ------------------------------------------------------------
R = 45.5e6                # Ohm
K0 = 39.5                 # N
omega = 9.55              # rad/s

# d11 = prefactor * M, where M is the slope of U versus f_lever in V
prefactor = np.pi / (2 * R * K0 * omega)

print("=" * 80)
print("Constants used")
print("=" * 80)
print(f"R         = {R:.3e} Ohm")
print(f"K0        = {K0:.3f} N")
print(f"omega     = {omega:.3f} rad/s")
```

```
print(f"prefactor = pi / (2 R K0 omega) = {prefactor:.6e} C/(N V)")
print()

# ------------------------------------------------------------
# 2. Common lever factors used everywhere
# ------------------------------------------------------------
x_values = np.array([30, 70, 110, 150, 190, 230, 270, 310, 350, 390],
    dtype=float)
f_lever = np.array([7.80, 5.57, 4.33, 3.55, 3.00, 2.60, 2.29, 2.05, 1.86,
    1.70], dtype=float)

# ------------------------------------------------------------
# 3. Raw voltage data from the report
#    IMPORTANT:
#    Quartz and unpolarized BT are in mV
#    Polarized BT is in V
# ------------------------------------------------------------

data = {
    "Quartz Crystal": {
        "unit": "mV",
        "temperatures": {
            24:  [240, 220, 180, 136, 120, 104,  92,  84,  76,  68],
            62:  [260, 200, 168, 136, 120,  96,  92,  84,  72,  64],
            99:  [240, 180, 160, 128, 112,  96,  88,  80,  72,  64],
            124: [250, 210, 170, 130, 110,  90,  80,  72,  64,  56],
        },
    },

    "Barium Titanate above Curie": {
        "unit": "mV",
        "temperatures": {
            135: [60, 60, 40, 40, 40, 20, 20, 10, 10, 5],
        },
    },

    "Barium Titanate - Polarized Cooling": {
        "unit": "V",
        "temperatures": {
            100: [11.2,  9.6, 7.2, 5.6, 4.0, 3.6, 3.0, 2.6, 2.2, 2.0],
             85: [ 5.2,  3.4, 2.8, 2.6, 2.4, 2.2, 1.44, 1.36, 1.20, 1.12],
```

```
            70: [ 5.2,  4.4, 4.0, 3.2, 2.6, 2.4, 2.2, 1.92, 1.60, 1.44],
            55: [ 5.2,  4.4, 3.4, 2.8, 2.4, 2.0, 1.92, 1.76, 1.36, 1.28],
        },
    },

    "Barium Titanate - Polarized Heating": {
        "unit": "V",
        "temperatures": {
            70: [5.6, 4.4, 4.0, 2.8, 2.4, 2.2, 2.0, 1.68, 1.60, 1.44],
            85: [6.8, 6.4, 4.8, 3.6, 3.2, 3.0, 2.6, 2.4, 1.92, 1.86],
           100: [9.6, 8.8, 7.2, 6.8, 6.0, 5.2, 4.8, 3.8, 3.2, 3.0],
        },
    },

    "Barium Titanate - Unpolarized Cooling": {
        "unit": "mV",
        "temperatures": {
           100: [440, 380, 340, 260, 220, 200, 180, 168, 152, 144],
            85: [420, 360, 300, 240, 200, 180, 140, 128, 112, 104],
            70: [400, 320, 240, 200, 180, 160, 136, 112, 104,  96],
            50: [360, 340, 240, 220, 168, 144, 128, 104,  96,  88],
        },
    },

    "Barium Titanate - Unpolarized Heating": {
        "unit": "mV",
        "temperatures": {
            50: [360, 340, 240, 220, 168, 144, 128, 104,  96,  88],
            70: [420, 380, 260, 220, 200, 160, 144, 136, 120, 112],
            85: [420, 360, 300, 240, 200, 180, 160, 140, 136, 120],
           100: [440, 400, 360, 220, 200, 184, 168, 160, 152, 144],
        },
    },
}

# ------------------------------------------------------------
# 4. Helper functions
# ------------------------------------------------------------
def linear_fit(x, y):
    """
    Fits y = m x + b and returns:
```

```
    slope m, intercept b, y_fit, R^2
    """
    coeffs = np.polyfit(x, y, 1)
    m, b = coeffs
    y_fit = m * x + b

    ss_res = np.sum((y - y_fit) ** 2)
    ss_tot = np.sum((y - np.mean(y)) ** 2)
    r_squared = 1 - ss_res / ss_tot if ss_tot != 0 else np.nan

    return m, b, y_fit, r_squared


def convert_slope_to_si(slope, unit):
    """
    Converts the slope into SI units (V per unit f_lever).
    """
    if unit == "V":
        return slope
    elif unit == "mV":
        return slope * 1e-3
    else:
        raise ValueError(f"Unknown voltage unit: {unit}")


def format_d11(d11_si):
    """
    Format d11 nicely in scientific notation.
    """
    return f"{d11_si:.3e} C/N"


def safe_filename(name):
    return (
        name.lower()
        .replace(" ", "_")
        .replace("-", "")
        .replace(":", "")
    )
```

```
# ------------------------------------------------------------
# 5. Main analysis loop
# ------------------------------------------------------------
all_results = {}

for scenario_name, scenario_data in data.items():
    unit = scenario_data["unit"]
    temperatures = scenario_data["temperatures"]

    print("=" * 80)
    print(scenario_name)
    print("=" * 80)
    print(f"Voltage unit in this scenario: {unit}")
    print()

    scenario_results = {}

    # ---- Plot setup for this scenario ----
    plt.figure(figsize=(9, 6.5))

    # Sort temperatures so the color gradient matches cold -> hot
    temps_sorted = sorted(temperatures.keys())

    # Colormap: blue -> violet/red
    cmap = cm.get_cmap("coolwarm")

    for i, temp in enumerate(temps_sorted):
        U_values = temperatures[temp]
        U = np.array(U_values, dtype=float)

        # Color for this temperature
        if  len(temps_sorted) == 1:
            color = "crimson"
        else:
            color = cmap(i / (len(temps_sorted) - 1))

        # Linear fit
        m, b, U_fit, r2 = linear_fit(f_lever, U)

        # Convert slope to SI and calculate d11
        m_si = convert_slope_to_si(m, unit)
```

```
        d11 = prefactor * m_si

        scenario_results[temp] = {
            "slope_raw": m,
            "intercept_raw": b,
            "slope_si": m_si,
            "r_squared": r2,
            "d11": d11,
        }

        # ---- Console output ----
        print(f"Temperature = {temp} deg C")
        print("-" * 60)
        print(f"Slope m              = {m:.6f} {unit}")
        print(f"Intercept b          = {b:.6f} {unit}")
        print(f"R^2                  = {r2:.6f}")
        print(f"Slope in SI units    = {m_si:.6e} V")
        print(f"d_11                 = {d11:.6e} C/N")
        print()

        # ---- Plot raw data + fit ----
        # Fit line in background
        plt.plot(
            f_lever, U_fit, "-",
            color=color,
            linewidth=2,
            alpha=0.9,
            zorder=1,
            label=f"{temp} C fit"
        )

        # Data points in foreground with black outline
        plt.plot(
            f_lever, U, "o",
            color=color,
            markersize=9,
            markeredgecolor="black",
            markeredgewidth=1.0,
            zorder=3,
            label=f"{temp} C data"
        )
```

```
244
245     # ---- Finish and save plot ----
246     plt.xlabel(r"$f_{\mathrm{lever}}$")
247     plt.ylabel(f"Voltage U ({unit})")
248     plt.title(scenario_name)
249     plt.grid(True)
250     plt.legend()
251     plt.tight_layout()
252
253     filename = f"plot_{safe_filename(scenario_name)}.png"
254     plt.savefig(filename, dpi=300, bbox_inches="tight")
255     plt.show()
256     plt.close()
257
258     print(f"Plot saved as: {filename}")
259     print()
260
261     all_results[scenario_name] = scenario_results
262
263 # ------------------------------------------------------------
264 # 6. Final compact summary for report writing
265 # ------------------------------------------------------------
266 print("=" * 80)
267 print("COMPACT SUMMARY")
268 print("=" * 80)
269
270 for scenario_name, scenario_results in all_results.items():
271     unit = data[scenario_name]["unit"]
272     print(f"\n{scenario_name}")
273     print("-" * 80)
274     print(f"{'T (C)':>8} {'m (' + unit + ')':>18} {'b (' + unit + ')':>18}
    {'R^2':>12} {'d_11 (C/N)':>18}")
275     print("-" * 80)
276
277     for temp in sorted(scenario_results.keys()):
278         res = scenario_results[temp]
279         print(
280             f"{temp:8.0f} "
281             f"{res['slope_raw']:18.6f} "
282             f"{res['intercept_raw']:18.6f} "
283             f"{res['r_squared']:12.6f} "
```

```
            f"{res['d11']:18.6e}"
        )


# ------------------------------------------------------------
# 7. Temperature dependence plots: d_11 versus T
# ------------------------------------------------------------

def extract_temperature_dependence(scenario_name):
    """
    Extract temperatures and d_11 values from all_results.
    Returns sorted arrays T and d11 in pC/N.
    """
    results = all_results[scenario_name]

    temperatures = np.array(sorted(results.keys()), dtype=float)
    d11_values = np.array([results[T]["d11"] for T in temperatures],
    dtype=float)

    # Convert C/N to pC/N for readability
    d11_pC_per_N = d11_values * 1e12

    return temperatures, d11_pC_per_N


def plot_d11_temperature_quartz():
    """
    Plot d_11 versus temperature for quartz.
    A horizontal mean line is added because quartz is expected to be only
    weakly
    temperature dependent.
    """
    T, d11 = extract_temperature_dependence("Quartz Crystal")

    d11_mean = np.mean(d11)
    d11_sem = np.std(d11, ddof=1) / np.sqrt(len(d11))

    plt.figure(figsize=(9, 6.5))

    plt.plot(
        T, d11,
```

```
        "o",
        markersize=9,
        markeredgecolor="black",
        markeredgewidth=1.0,
        label="Measured values"
    )

    plt.axhline(
        d11_mean,
        linestyle="--",
        linewidth=2,
        label=rf"Mean: {d11_mean:.2f} pC/N"
    )

    plt.fill_between(
        [T.min(), T.max()],
        d11_mean - d11_sem,
        d11_mean + d11_sem,
        alpha=0.15,
        label=rf"SEM: $\pm$ {d11_sem:.2f} pC/N"
    )

    plt.xlabel(r"Temperature $T$ ($^\circ$C)")
    plt.ylabel(r"$d_{11}$ (pC/N)")
    plt.title(r"Quartz Crystal: $d_{11}$ as a Function of Temperature")
    plt.grid(True, alpha=0.4)
    plt.legend()
    plt.tight_layout()

    filename = "plot_d11_temperature_quartz.png"
    plt.savefig(filename, dpi=300, bbox_inches="tight")
    plt.show()
    plt.close()

    print(f"Temperature dependence plot saved as: {filename}")


def plot_d11_temperature_heating_cooling(cooling_name, heating_name, title,
    filename):
    """
    Plot d_11 versus temperature for cooling and heating.
```

```
    Points are slightly shifted horizontally so overlapping values remain visible.
    No trendline is fitted, because the temperature dependence is not expected to be linear.
    """
    T_cooling, d11_cooling = extract_temperature_dependence(cooling_name)
    T_heating, d11_heating = extract_temperature_dependence(heating_name)

    # Small horizontal offset only for visibility
    offset = 0.3

    plt.figure(figsize=(9, 6.5))

    plt.plot(
        T_cooling - offset,
        d11_cooling,
        "o",
        markersize=9,
        markeredgecolor="black",
        markeredgewidth=1.0,
        label="Cooling"
    )

    plt.plot(
        T_heating + offset,
        d11_heating,
        "s",
        markersize=9,
        markeredgecolor="black",
        markeredgewidth=1.0,
        label="Heating"
    )

    plt.xlabel(r"Temperature $T$ ($^\circ$C)")
    plt.ylabel(r"$d_{11}$ (pC/N)")
    plt.title(title)
    plt.grid(True, alpha=0.4)
    plt.legend()
    plt.tight_layout()

    plt.savefig(filename, dpi=300, bbox_inches="tight")
```

```
    plt.show()
    plt.close()

    print(f"Temperature dependence plot saved as: {filename}")


# ---- Quartz ----
plot_d11_temperature_quartz()


# ---- Barium titanate: polarized sample ----
plot_d11_temperature_heating_cooling(
    cooling_name="Barium Titanate - Polarized Cooling",
    heating_name="Barium Titanate - Polarized Heating",
    title=r"Barium Titanate Polarized: $d_{11}$ as a Function of Temperature",
    filename="plot_d11_temperature_batio3_polarized.png"
)


# ---- Barium titanate: unpolarized sample ----
plot_d11_temperature_heating_cooling(
    cooling_name="Barium Titanate - Unpolarized Cooling",
    heating_name="Barium Titanate - Unpolarized Heating",
    title=r"Barium Titanate Unpolarized: $d_{11}$ as a Function of
   Temperature",
    filename="plot_d11_temperature_batio3_unpolarized.png"
)
```